ASTR3002 ASSIGNMENT 2

Galactic Dynamics

Due October 29, 2007

You can send your assignments to me through the internal mail (Prof K.C. Freeman, Research School of Astronomy & Astrophysics). Please don't be late. Remember – the two assignments are your only formal assessment for this part of the course

1. A spherical galaxy has a density distribution

$$\rho(r) = \rho_o[1 + r^2/a^2]^{-1}.$$

Show that the enclosed mass $M(r) \propto r^3$ for $r \ll a$ (*i.e.* in the core) and $M(r) \propto r$ for $r \gg a$ (*i.e.* in the outer halo).

Now take a population of massless test particles in the potential of this galaxy. Assume that this population is spherical, non-rotating, isothermal and isotropic, with velocity dispersion σ in each velocity component. What is the radial density distribution of this test particle population ?

At large r, this density distribution simplifies and its form depends on a dimensionless number. Give a physical interpretation of this dimensionless number. What is the condition that the density distributions of the test particle population and the galaxy itself have similar forms at large r ? (20 marks for whole question)

2. (B&T 4.6). Suppose the local mass density of matter in the galactic disk near the sun has been estimated by using some tracer population whose distance has been systematically overestimated by a factor λ. By what factor is the derived local mass density in error ? (10 marks)

3. (B&T 4.11). The velocity dispersion tensor $\boldsymbol{\sigma}$ in some axisymmetric, time-independent stellar system is isotropic and a function $\sigma(\rho)$ of the density alone; *i.e.* σ depends on (R, z) only through the density $\rho(R, z)$ (like a barotropic fluid). Show that the azimuthal streaming velocity $\overline{v}_\phi$ is a function $\overline{v}_\phi(R)$ of the cylindrical radius R only. Is such a system physically plausible ? Give a brief qualitative description of the typical stellar orbits in such a system. (10 marks)

4. This is a simple question on selfconsistent systems. We want to make a *uniform density*, axisymmetric infinite cylinder of radius D and density ρ, using a distribution function of the form $f = f(E)$. The axis of symmetry of the cylinder is in the z-direction. The potential of such a cylinder has the form $\Phi(R) = \kappa R^2$ where R is the usual (cylindrical) radial distance from the axis of symmetry and κ is a constant. Then

$$\rho = \int f(E) dv_R dv_\phi,$$

where we have already integrated over v_z.

(a) Find the relationship between κ and ρ

(b) Find $f(E)$

(c) Calculate the velocity dispersion components in the (R, ϕ) plane.

Hint: think carefully about the limits of integration in the integrals over $dv_R dv_\phi$ – everything else is easy. (20 marks for whole question).

5. This is an essay question. Write an essay of about 500 words comparing and contrasting the fundamental plane relation for elliptical galaxies with the Tully-Fisher relation for disk galaxies. Try to include some information that is *not* in the notes that I handed out on elliptical and disk galaxies. (20 marks)