

## ASTR3002 ASSIGNMENT 2

### Galactic Dynamics

Due August 31, 2006

You can send your assignments to me through the internal mail (Prof K.C. Freeman, Research School of Astronomy & Astrophysics). Please don't be late. They must reach me by August 31, 2006. Remember – the two assignments are your only formal assessment for this part of the course

1. A spherical galaxy has a density distribution

$$\rho(r) = \rho_0[1 + r^2/a^2]^{-1}.$$

Show that the enclosed mass  $M(r) \propto r^3$  for  $r \ll a$  (*i.e.* in the core) and  $M(r) \propto r$  for  $r \gg a$  (*i.e.* in the outer halo).

Now take a population of massless test particles in the potential of this galaxy. Assume that this population is spherical, non-rotating, isothermal and isotropic, with velocity dispersion  $\sigma$  in each velocity component. What is the radial density distribution of this test particle population ?

At large  $r$ , this density distribution simplifies and its form depends on a dimensionless number. Give a physical interpretation of this dimensionless number. What is the condition that the density distributions of the test particle population and the galaxy itself have similar forms at large  $r$  ? (20 marks for whole question)

2. (B&T 4.6). Suppose the local mass density of matter in the galactic disk near the sun has been estimated by using some tracer population whose distance has been systematically overestimated by a factor  $\lambda$ . By what factor is the derived local mass density in error ? (10 marks)

**3.** (B&T 4.11). The velocity dispersion tensor  $\sigma$  in some axisymmetric, time-independent stellar system is isotropic and a function  $\sigma(\rho)$  of the density alone; *i.e.*  $\sigma$  depends on  $(R, z)$  only through the density  $\rho(R, z)$ . Show that the azimuthal streaming velocity  $\bar{v}_\phi$  is a function  $\bar{v}_\phi(R)$  of the cylindrical radius  $R$  only. Is such a system physically plausible? Give a brief qualitative description of the typical stellar orbits in such a system. (10 marks)

**4.** This is a simple question on selfconsistent systems. We want to make a *uniform density*, axisymmetric infinite cylinder of radius  $D$  and density  $\rho$ , using a distribution function of the form  $f = f(E)$ . The axis of symmetry of the cylinder is in the  $z$ -direction. The potential of such a cylinder has the form  $\Phi(R) = \kappa R^2$  where  $R$  is the usual (cylindrical) radial distance from the axis of symmetry and  $\kappa$  is a constant. Then

$$\rho = \int f(E) dv_R dv_\phi,$$

where we have already integrated over  $v_z$ .

(a) Find the relationship between  $\kappa$  and  $\rho$

(b) Find  $f(E)$

(c) Calculate the velocity dispersion components in the  $(R, \phi)$  plane.

Hint: think carefully about the limits of integration in the integrals over  $dv_R dv_\phi$  – everything else is easy. (20 marks for whole question).

**5.** This is an essay question. Write an essay of about 500 words comparing and contrasting the fundamental plane relation for elliptical galaxies with the Tully-Fisher relation for disk galaxies. Try to include some information that is *not* in the notes that I handed out on elliptical and disk galaxies. (20 marks)