ASTR3002 ASSIGNMENT 1

Galactic Dynamics

Due August 14, 2006

You can send your assignments to me through the internal mail (Prof K.C. Freeman, Research School of Astronomy & Astrophysics). or give them to me at lectures. Please don't be late: remember – the two assignments are your only formal assessment for this course

1. Give a simple order of magnitude argument to show why the relaxation time for 2-body encounters in a stellar system $T_R \sim \sigma^3/m^2 n$, where σ is the velocity dispersion, m the typical stellar mass and n the number density of stars. (10 marks)

2. (From B&T 99, question 2.3). The density distribution

$$\rho(r) = \frac{M}{4\pi r_J^3} \frac{r_J^4}{r^2(r + r_J)^2}$$

(where M and r_J are constants) is called a Jaffe model (after Walter Jaffe of Leiden, who devised it). Show that its potential is

$$\Phi = \frac{GM}{r_J} \ln(\frac{r}{r + r_J}).$$

Verify that the total mass is M. Show that the circular speed v_c is approximately constant for $r \ll r_J$ and falls off like $v_c \sim r^{-1/2}$ for $r \gg r_J$: why could you have foreseen this from just looking at the density distribution ? Comment on Jaffe models as models for real galaxies. (See *Useful Results from Potential Theory* in lecture notes.) (10 marks)

3. Consider a distant spherical galaxy with potential

$$\Phi(r) = V^2 \ln(r)$$

in which the stellar orbits are all circular but randomly orientated. What is the probability distribution of the line-of-sight component of the velocities of the stars in this galaxy ? (Suggestion: work out the probability distributions of the orbital phase angle and of the angle between the line of sight and the normal to the circular orbits). You can do this question analytically or numerically or both. (20 marks).

4. (B&T 4.17). For a Maxwellian distribution of velocities with one-dimensional velocity dispersion σ, the number of stars with velocities in some element d^3v about velocity v is

$$dN = F(v)d^3v = \text{const.}\exp(-v^2/2\sigma^2)d^3v.$$

Show that:

(a) the mean speed $\overline{v} = \sqrt{8/\pi}\ \sigma$

(b) the mean square speed $\overline{v^2} = 3\sigma^2$

(c) the mean square of one component of velocity is (say) $\overline{v_x{}^2} = \sigma^2$

(d) the fraction of stars with $v^2 > 4\overline{v^2}$ is 0.00738
(10 marks).

5.a) In a spherical galaxy with potential

$$\Phi(r) = V^2 \ln(r)$$

with $V = 200$ km s^{-1}, a star at $r = 10$ kpc has $v_t = 100$ km s^{-1} , $v_r = 50$ km s^{-1}, where v_t and v_r are the tangential and radial components of the stellar velocity. What are the maximum and minimum values of r for the star's orbit ? (Hint: energy and angular momentum are conserved).

5.b) Integrate this orbit numerically until it reaches its third apogalacticon (maximum r).

- plot the orbit in its own plane
- plot $r(t)$
- what is the angular distance between successive apogalactica
- check that the apo- and perigalactic distances agree with your estimates in **5.a)**.

(Suggestion: make the equations dimensionless: write $\dot{r}' = \dot{r}/V$, $r' = r/10$ kpc etc.)

5.c) In this same potential, sketch the following in the (E, J) plane, where $E = v^2/2 + \Phi$, $J = rv_t$ are the energy and angular momentum of a star: *(i)* loci of circular orbits, *(ii)* a few loci of constant apogalactica, *(iii)* a few loci of constant perigalactica.

Say we allow no orbits with $E > E_\circ$, where $E_\circ$ is some constant. What is the ratio of the apogalactic radii for circular and radial orbits with $E = E_\circ$? Mark these two orbits on your (E, J) plane. What do you conclude about the properties of orbits in the outer parts of stellar systems with a sharp energy cutoff ? (20 marks for whole question).