

High Energy Astrophysics

Solutions to Questions 1-4

1. The relationship between energy and wavelength is

$$\varepsilon = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3.0 \times 10^8}{1.60 \times 10^{-19} \times \left(\frac{\lambda}{\text{m}}\right)} = 1.24 \times 10^{-6} \left(\frac{\lambda}{\text{m}}\right)^{-1} \text{ eV}$$

Also, to determine frequency and wavelength given energy, use

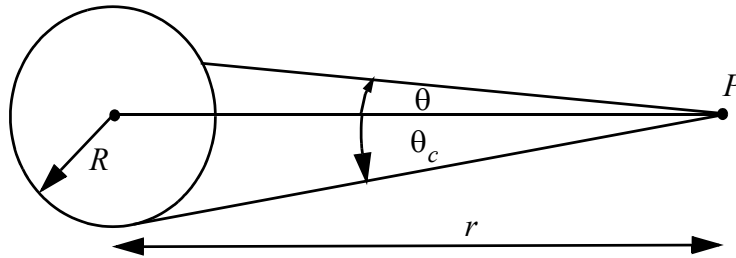
$$\varepsilon = h\nu \Rightarrow \nu = \frac{\varepsilon}{h} = \frac{1.60 \times 10^{-19}}{6.62 \times 10^{-34}} \times \left(\frac{\varepsilon}{\text{eV}}\right) = 2.41 \times 10^{14} \left(\frac{\varepsilon}{\text{eV}}\right) \text{ Hz}$$

$$\varepsilon = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\varepsilon} = 1.24 \times 10^{-6} \left(\frac{\varepsilon}{\text{eV}}\right)^{-1} \text{ m} = 1.24 \times 10^4 \left(\frac{\varepsilon}{\text{eV}}\right)^{-1} \text{ Angstroms}$$

Hence the following table (the primary quantity is given in bold):

| Photon | ν (Hz) | λ (Angstroms) | λ (metres) | ε (eV) |
|-----------------------|-----------------------|--------------------------|---|-----------------------|
| Typical microwave | | | 0.20 | 6.2×10^{-6} |
| Typical far infrared | | | 100×10^{-6} $= 10^{-4}$ | 1.1×10^{-2} |
| Typical near infrared | | | 2×10^{-6} | 0.62 |
| Typical optical | | 5000 | 5×10^{-7} | 2.48 |
| Typical UV | | 1000 | 10^{-7} | 12.4 |
| Soft X-ray | 2.41×10^{17} | 12.4 | 1.24×10^{-9} | 1 keV |
| Hard X-ray | 2.41×10^{19} | 0.124 | 1.24×10^{-11} | 100 keV |
| TeV γ -ray | 2.41×10^{26} | 1.24×10^{-8} | 1.24×10^{-18} | 1 TeV |

2.

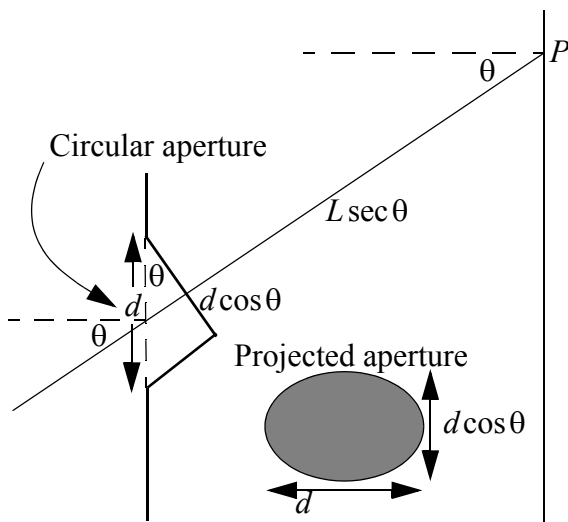
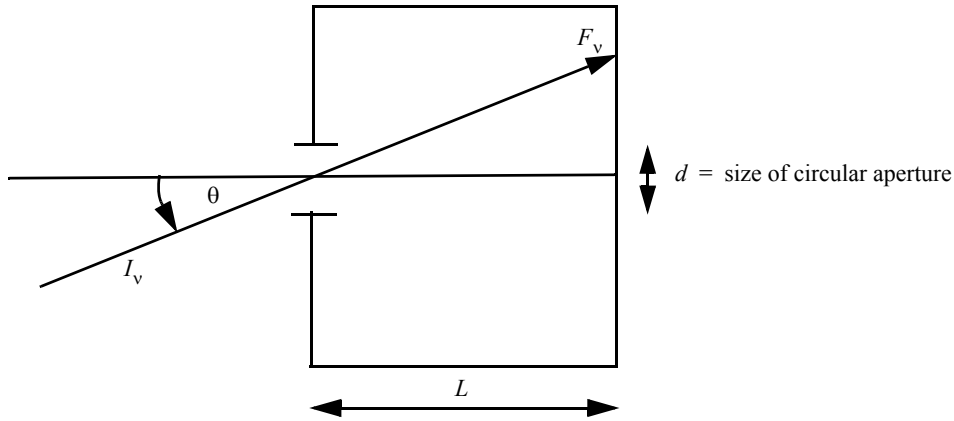


The flux density received at point P is:

$$\begin{aligned} F_v &= \int_{\Omega} I_v \cos \theta d\Omega = \int_{\theta, \phi} I_v \cos \theta \sin \theta d\theta d\phi \\ &= 2\pi I_v \int_0^{\theta_c} \cos \theta \sin \theta d\theta = \pi I_v \sin^2 \theta_c \\ &= \pi I_v \frac{R^2}{r^2} \end{aligned}$$

That is, we recover the inverse square law.

3.



First calculate the solid angle of rays hitting the back of the camera. Viewed from the point P the aperture appears as an ellipse with major and minor axes d and $d \cos \theta$. The area of the aperture is therefore

$$\Delta A \approx \frac{\pi}{4} d \cos \theta \times d = \frac{\pi}{4} d^2 \cos \theta$$

Hence the (small) solid angle subtended by the aperture at P is

$$\Delta \Omega = \frac{\frac{\pi}{4} d^2 \cos \theta}{(L \sec \theta)^2} = \frac{\pi d^2}{4 L^2} \cos^3 \theta$$

Next, the flux density incident on the back of the camera at P is

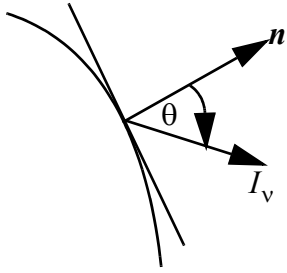
$$F_v = \int_{\Omega} I_v \cos \theta d\Omega \approx I_v \cos \theta \Delta \Omega = I_v \frac{\pi d^2}{4 L^2} \cos^4 \theta = I_v \frac{\frac{\pi}{4} \cos^4 \theta}{f^2}$$

where the focal ratio $f = L/d$.

4. (a) The mean intensity at a particular point, a radius r from the centre of the sphere is:

$$J_v = \frac{1}{4\pi} \int_{\Omega} I_v d\Omega = \frac{1}{2} \int_0^{\theta_c} I_v \sin \theta d\theta = \frac{1}{2} I_v [1 - \cos \theta_c] = \frac{1}{2} I_v \left[1 - \left(1 - \frac{R^2}{r^2} \right)^{1/2} \right]$$

(b) Flux through the surface of the sphere is:



$$F_v = \int_0^{2\pi} \left\{ \int_0^{\pi} I_v \cos\theta \sin\theta d\theta \right\} d\phi = \pi I_v$$

Hence, the total energy per unit time through the surface of the sphere (i.e. the Luminosity) is:

$$L_v = F_v \times 4\pi R^2 = 4\pi^2 I_v R^2$$

Therefore,

$$I_v = \frac{L_v}{4\pi^2 R^2}$$

(c) The energy density per unit frequency is

$$\begin{aligned} u_v &= \frac{4\pi}{c} J_v = \frac{4\pi}{c} \times \frac{1}{2} I_v \left[1 - \left(1 - \frac{R^2}{r^2} \right)^{1/2} \right] \\ &= \frac{L_v}{2\pi c R^2} \left[1 - \left(1 - \frac{R^2}{r^2} \right)^{1/2} \right] \\ &\approx \frac{L_v}{4\pi c R^2} \end{aligned}$$

Hence, the total (i.e. frequency integrated) energy density is

$$u \approx \frac{L}{4\pi c R^2}$$

(d) This expression can also be derived as follows. The total energy passing through a sphere of radius r is

$$u \times c \times 4\pi r^2$$

and this is equal to the total luminosity, L . Hence

$$u \times c \times 4\pi r^2 = L \Rightarrow u = \frac{L}{4\pi c r^2}$$

(e) For $L = 3.83 \times 10^{26}$ W and $r = 1.50 \times 10^{11}$ m

$$u = 4.5 \times 10^{-6} \text{ J m}^{-3} = 2.8 \times 10^{13} \text{ eV m}^{-3}$$