

# Definitions of Magnitudes and Surface Brightness

## 1 The magnitude system

### 1.1 Apparent magnitude

Astronomers are stuck with the antiquated old system of magnitudes and so a clear explanation of what this system is and how it relates to more sensible units is important.

The magnitude scale is a logarithmic scale which in the first place related to stars. The larger the magnitude the fainter a star and the system is defined so that 5 magnitudes corresponds to a factor of 100 in flux. Let  $F$  be the broad band flux (i.e. the flux integrated through some filter), and let  $F_0$  be the flux which corresponds to zero apparent magnitude, then

$$m = -2.5 \log\left(\frac{F}{F_0}\right)$$

You can see from this expression that if  $F = F_0$  then  $m = 0$  and that if  $F$  decreases by a factor of 100 then  $m$  increases by 5.

The inverse relationship is

$$F = F_0 \times 10^{-0.4m}$$

In order to convert magnitudes to flux one needs to know  $F_0$ , the “flux for zero magnitude”. There are tables of this for various filters.

## 1.2 Absolute magnitude

Absolute magnitude is a logarithmic luminosity scale. Formally absolute magnitude is defined by:

*The absolute magnitude of a star is the apparent magnitude that it would have if it were located at a distance of 10 pc.*

Suppose then that a star is located at a distance of  $D$  pc and that the flux received from the star is  $F$   $\text{W m}^{-2}$ . Then the flux received from 10 pc would be

$$F_{10} = F \times \left(\frac{D}{10}\right)^2$$

The absolute magnitude is then:

$$M = -2.5 \log\left(\frac{F_{10}}{F_0}\right) = -2.5 \log\left(\frac{FD^2}{F_0 \times 100}\right) = -2.5 \log \frac{F}{F_0} - 5 \log D + 5$$

so that the relationship between apparent and absolute magnitude is:

$$M = m - 5 \log D + 5$$

where one has to remember that  $D$  is expressed in parsecs.

### 1.3 Conversion to absolute units

One often does not convert absolute magnitudes directly to  $\text{W m}^{-2}$ . One usually goes by the round about process of referring to the absolute magnitude of the sun. In the B and V bands for example, this is:

$$M_B = 5.46 \quad M_V = 4.84$$

and the bolometric absolute magnitude of the sun (obtained from integrating across the entire spectrum) is:

$$M_{\text{bol}} = 4.77$$

Since absolute magnitude is defined by

$$M = -2.5 \log \left( \frac{F_{10}}{F_0} \right)$$

then we have for the sun:

$$M_{\text{sun}} = -2.5 \log \left( \frac{F_{10, \text{sun}}}{F_0} \right)$$

and the difference between the absolute magnitude of a star and the absolute magnitude of the sun is given by:

$$M - M_{\text{sun}} = -2.5 \log \left( \frac{F_{10}}{F_{10, \text{sun}}} \right) = -2.5 \log \left( \frac{L}{L_{\text{sun}}} \right)$$

The luminosity of the star compared to the luminosity of the sun is given by:

$$\frac{L}{L_{\text{sun}}} = 10^{-0.4(M - M_{\text{sun}})}$$

The  $L_{\text{sun}}$  here is the luminosity of the sun in that particular band. This can be evaluated from the absolute magnitude of the sun in that band, the bolometric magnitude and the bolometric luminosity of the sun,  $L_{\text{bol}} = 3.38 \times 10^{26}$  W. Hence,

$$\frac{L_{\text{sun}}}{L_{\text{bol}}} = 10^{-0.4(M_{\text{sun}} - M_{\text{bol}})}$$

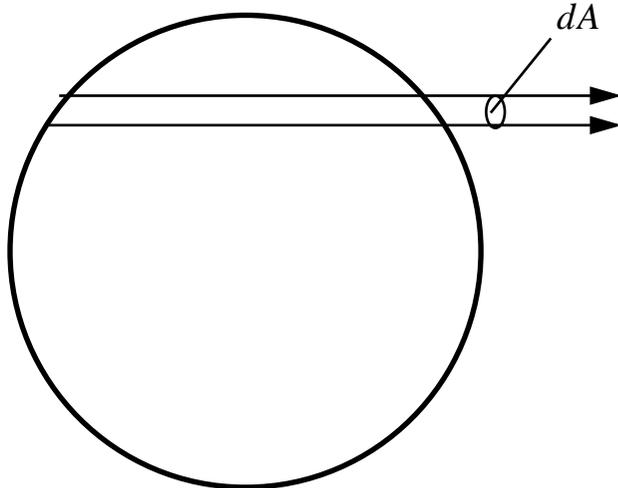
## 2 Surface brightness of galaxies and plasma

Astronomers measure the surface brightness of galaxies, the surface brightness of radiation from stars and the surface brightness of radio sources. Unfortunately the definition of the first does not coincide with the second two.

In the context of the radiation field, the surface brightness of an optically thin source is given by

$$I = \int j ds$$

where  $j$  is some broad-band (integrated) emissivity, whose units are  $\text{Wm}^{-3}\text{Sr}^{-1}$  and  $s$  is the path length through the source. That is,  $j$  is integrated power per unit solid angle.



Galactic dynamicists take a slightly different approach. They define surface brightness in terms of the total luminosity emitted by a column of linear area  $dA$  (Area in this case is often measured in units of parsecs.)

Let  $l$  be the luminosity density at an arbitrary point within the galaxy, i.e.  $l$  is the power emitted per unit volume in all directions. The emission is isotropic and therefore no solid angle dependence is implied. The total luminosity emitted by a column with area  $dA$  is

$$dL = dA \int l ds$$

Hence,

$$\Sigma = \frac{dL}{dA} = \int l ds$$

The quantity  $\frac{dL}{dA}$  has the physical units of  $\text{W m}^{-2}$  and in galaxy studies is often represented in solar luminosities per square parsec and is given the symbol  $\Sigma$  - and is also called *surface brightness*.

The reason for this definition lies in the origin of the magnitude system. For a given patch of galaxy, astronomers determine a magnitude and then, using the relations between absolute magnitude and solar luminosity convert this to a luminosity expressed in solar luminosities.

The luminosity density  $l$  and the emissivity  $j$  are related by:

$$j = \frac{1}{4\pi} l$$

so that

$$\Sigma = 4\pi \int j ds = 4\pi I$$

$$\Rightarrow I = \frac{1}{4\pi} \Sigma$$

Hence, in calculating quantities which are relevant to the radiation field one has to divide the galaxy surface brightness by  $4\pi$ , making sure to express  $\Sigma$  in appropriate physical units.