High Energy Astrophysics

Solutions to Assignment 5

18 (a) Again we can use the results of Q. 3. We have

$$u_\nu(r_0) = \frac{4\pi}{c} J_\nu(r_0) = \frac{4\pi}{c}\int_0^\infty j_\nu(r) K\left(\frac{r}{r_0}\right) dr$$

In this case $j_\nu = \text{constant}$ so that:

$$u_\nu(r_0) = \frac{4\pi j_\nu r_0}{c}\int_0^{R/r_0} K\left(\frac{r}{r_0}\right) d\left(\frac{r}{r_0}\right) = \frac{4\pi j_\nu r_0}{c}\int_0^{R/r_0} K(t)\,dt$$

Using Maple,

$$\int_0^{R/r_0} K(t)\,dt = \frac{1}{4}\left[\left(\frac{R}{r_0}\right)^2 - 1\right]\ln\left[\frac{\frac{R}{r_0}+1}{\frac{R}{r_0}-1}\right] + \frac{1}{2}\frac{R}{r_0}$$

and

$$u_\nu(r_0) = \frac{4\pi j_\nu R}{c}\left[\frac{1}{4}\left(\frac{R}{r_0} - \frac{r_0}{R}\right)\ln\left(\left(\frac{1 + r_0/R}{1 - r_0/R}\right) + \frac{1}{2}\right)\right]$$

That is, putting $\xi = r/R$,

$$\begin{aligned} u_\nu(r) &= \left[\frac{4\pi j_\nu R}{c}\right]\left[\frac{1}{4}(\xi^{-1} - \xi)\ln\left(\frac{1+\xi}{1-\xi}\right) + \frac{1}{2}\right] \\ &= \frac{\pi I_\nu^{\text{peak}}}{c}\left[\frac{1}{2}(\xi^{-1} - \xi)\ln\left(\frac{1+\xi}{1-\xi}\right) + 1\right] \end{aligned}$$

since the peak intensity through the blob is given by

$$I_\nu^{\text{peak}} = j_\nu \times 2R$$

(b) Since $\ln(1+x) \approx x$ for x small, we can expand the expression in square brackets to obtain

$$\frac{1}{4}(\xi^{-1} - \xi)\ln\left(\frac{1+\xi}{1-\xi}\right) + \frac{1}{2} = 1 + O(\xi)$$

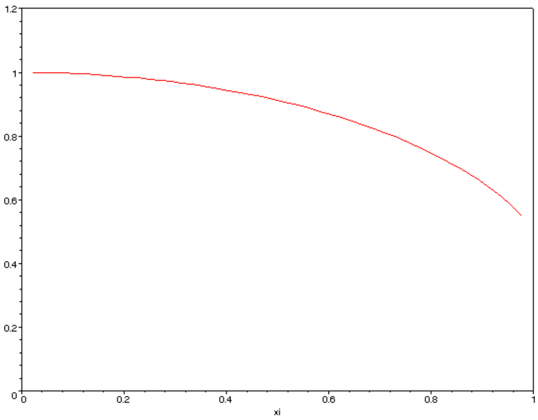
so that

$$u_\nu(0) = \frac{4\pi j_\nu R}{c}$$

The mean intensity at the centre of the blob is $j_\nu R$ so that this equation reads

$$u_\nu(0) = \frac{4\pi}{c} J_\nu(0)$$

and is therefore consistent.



(c) A plot of the function of ξ is shown at the left. An interesting feature of this function is that is does not vary a great deal over the range of ξ from 0 to 1.

19. (a) In estimating the radiation energy density of the hot spot we assume that it is spherical.In this case the hot spot is not as aspherical as it might seem because the beam is elongated in the N-S direction.

The central radiation energy density per unit frequency is given by:

$$u_\nu = \frac{2\pi I_\nu^{\text{peak}}}{c}$$

Hence the total radiation energy density is given by

$$u = \frac{2\pi}{c} \int_{\nu_l}^{\nu_u} I_{\nu_0} \left(\frac{\nu}{\nu_0}\right)^{-\alpha} d\nu = \frac{2\pi I_{\nu_0} \nu_0}{c(1-\alpha)} \left[\left(\frac{\nu}{\nu_0}\right)^{1-\alpha}\right]_{\nu_l}^{\nu_u}$$

The lower frequency does not make much difference to the numerical answer. To estimate the radiation energy density, we use

$$\nu_0 = \frac{3\times10^8}{0.02} = 1.5\times10^{10} \qquad I_{\nu_0} = \frac{58.8\times10^{-29}}{2.66\times10^{-11} \times 0.45 \times 0.09} \text{W m}^{-2}\text{Hz Sr}^{-1}$$

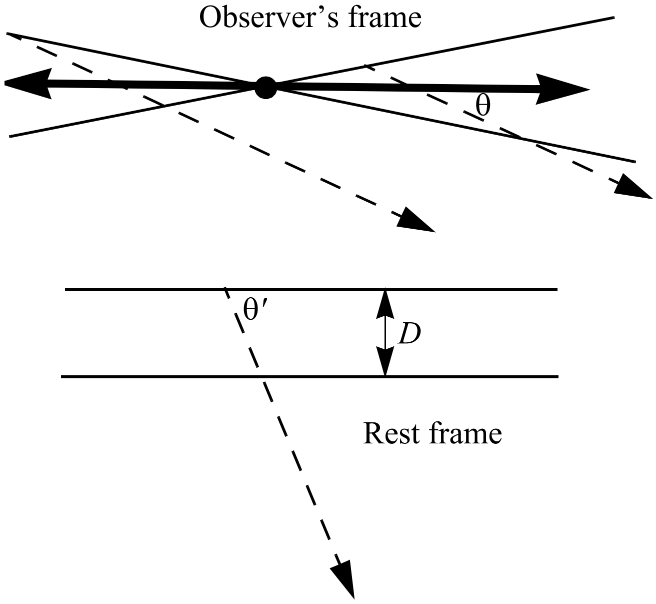
$$\nu_u = 10^{14}\ \text{Hz} \qquad \nu_l \approx 0$$

The result is:

$$u \approx 8.0\times10^{-12}\ \text{J m}^{-3}$$

(b) The energy density of the microwave background is approximately 4.2×10^{-15} J m^{-3} so that the energy density from synchrotron radiation in the Pictor A hotspot is approximately a factor of 1.9×10^3 larger.

Combining the terms with the same phase:



22. Consider the surface brightness in the rest frame (dashed symbols) of the jet. This gives

$$I_{\nu'}' = j_{\nu'}\left(\frac{D}{\sin\theta'}\right)$$

Using the Lorentz transformation for the surface brightness

$$\frac{I_\nu}{\nu^3} = \frac{I_{\nu'}}{(\nu')^3} \Rightarrow I_\nu = \left(\frac{\nu}{\nu'}\right)^3 I_{\nu'} = \delta^3 I_{\nu'}'$$

Also

$$\sin\theta' = \frac{\sin\theta}{\Gamma(1-\beta\cos\theta)} = \delta\sin\theta$$

Therefore,

$$I_\nu = \delta^3 j_{\nu'}\frac{D}{\delta\sin\theta} = \delta^2 j_{\nu'}\frac{D}{\sin\theta}$$

Now normalise the emissivity to the frequency of observation.

$$j_{\nu'} = j_\nu\left(\frac{\nu'}{\nu}\right)^{-\alpha} = j_\nu\delta^\alpha$$

giving

$$I_\nu = \delta^{2+\alpha} j_\nu\left(\frac{D}{\sin\theta}\right)$$

Note that if $|\beta|$ is the magnitude of β, then

$\delta = \dfrac{1}{\Gamma(1+|\beta|\cos\theta)}$ for the receding jet and $\delta = \dfrac{1}{\Gamma(1-|\beta|\cos\theta)}$ for the approaching jet.

Hence the ratio of jet to counterjet surface brightness is

$$R = \left[\frac{\Gamma(1+\beta\cos\theta)}{\Gamma(1-\beta\cos\theta)}\right]^{2+\alpha} = \left[\frac{1+\beta\cos\theta}{1-\beta\cos\theta}\right]^{2+\alpha}$$

(ii) Solving for $\beta\cos\theta$ gives

$$\beta\cos\theta = \frac{R^{\frac{1}{2+\alpha}}-1}{R^{\frac{1}{2+\alpha}}+1}$$

(iii) In M87, $R > 150$ and $\alpha = 1.5$. Hence,

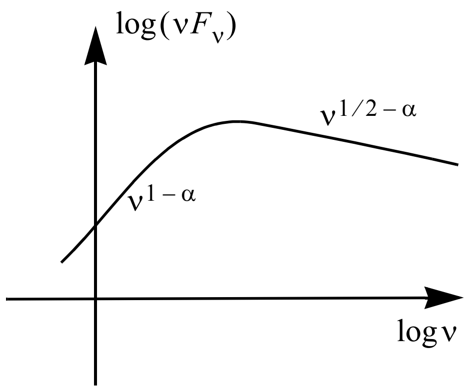
$$\beta\cos\theta > \frac{150^{1/3.5}-1}{150^{1/3.5}+1} = 0.61$$

Therefore

$$\beta > \frac{0.61}{\cos\theta} > 0.61$$

This is the minimum velocity of the jet.
The largest inclination angle is given by:

$$\cos\theta > \frac{0.61}{\beta} > 0.61 \Rightarrow \theta < \cos^{-1}0.61 = 52°$$



23. The main difference between this example and the one given in the text is that the extrapolation of the synchrotron emission to the X-ray could well make an important contribution in that region of the spectrum. Diagrammatically this is represented by the spectral plot on the left. When a population of synchrotron emitting electrons is subject to both cooling and escape from a given region a break in the electron spectrum occurs that is reflected in the integrated emission from that region. Note that the change in spectral index is $1/2$ and that the "broken spectrum" is quite flat in νF_ν so that the contribution to high energies can be quite substantial.
Therefore, in estimating the contribution to νF_ν from the SSC emission one has to subtract off the extrapolated synchrotron component.
(i) My spreadsheet for this calculation is Pictor_A.xls and is on the course web-site.
The value of the SSC-inferred magnetic field is 1.5×10^{-5} G (see sheet 2 of the spreadsheet).
(ii) The value of the minimum energy magnetic field is 1.9×10^{-4} G (see sheet 1). The SSC-estimated magnetic field is therefore about an order of magnitude smaller than the minimum energy magnetic field, highlighting the necessity for caution in using minimum energy estimates.
(iii) One of the intriguing features of the Pictor A X-ray spectrum is that the slope of the spectrum is rather flat and significantly different from the radio slope in the region before the break. One possibility is that the slope of the X-ray mirrors the slope of the synchrotron component near the break in spectral index. If this is the case then the frequency of the X-ray photons would be given by:

$$\nu_X = \gamma_b^2 \nu_b$$

where γ_b is the Lorentz factor of electrons radiating near the break and ν_b is the break frequency. Now the break frequency is given by:

$$\nu_b \sim \frac{3}{4\pi}\left(\frac{eB}{m_e}\right)\gamma^2 = \frac{3}{4\pi}\left(\frac{eB}{m_e}\right)\frac{\nu_X}{\nu_b} \Rightarrow B = \frac{4\pi}{3}\frac{m_e}{e}\frac{\nu_b^2}{\nu_X}$$

Take $\nu_b = 10^{14}$ and $\nu_X = 10^{17.5}$. This gives $B = 0.75\ \mathrm{T} = 7.5\times10^{3}$ G – an outrageously large magnetic field.

The only way that such an idea would work would be if the radio spectrum had a break at a much lower frequency. In order to obtain a magnetic field comparable to the minimum energy value $\sim 10^{-4}$ G, one requires a break frequency about a factor of 10^{4} lower, i.e. at about 10^{10} Hz. This is the putative low energy component referred to by Wilson et al.