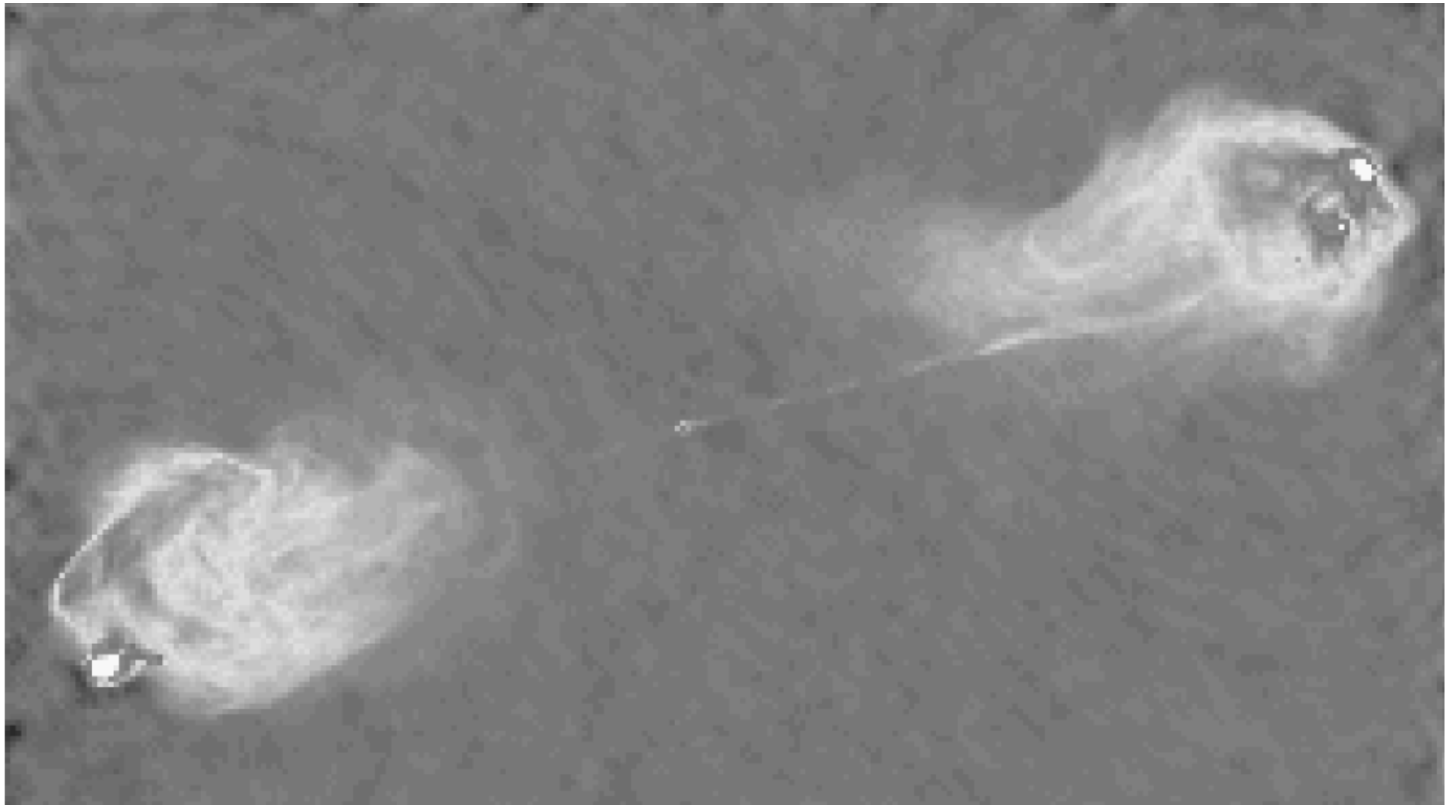
Astrophysical Applications of Relativity

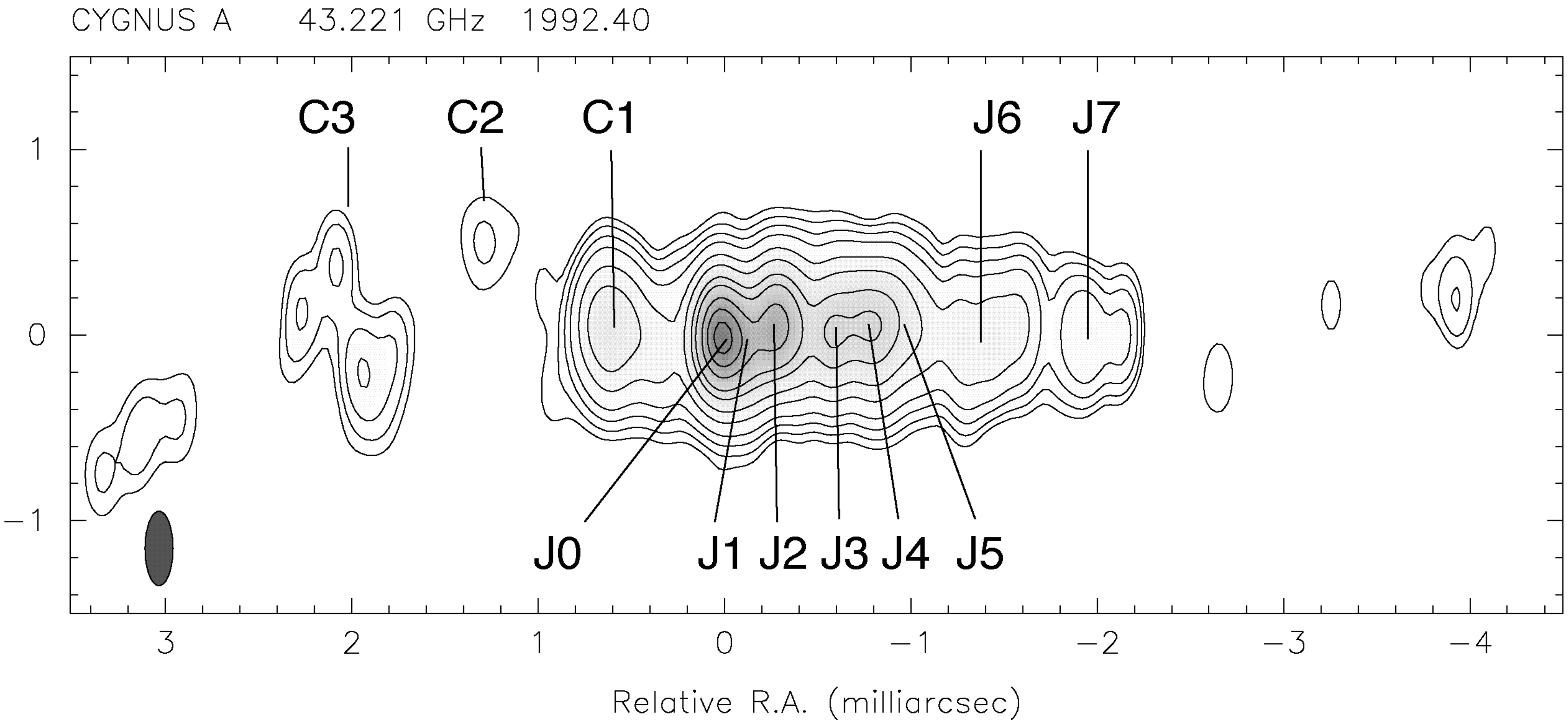
1 Examples of relativistic fluids



The powerful radio source Cygnus A, ($z = 0.056075$)

(Carilli, Perley, Bartel and Dreher)

See Cygnus A - Study of a radio galaxy, CUP, ed. Carilli and Harris.



43 GHz VLBI image of the core of Cygnus A (Krichbaum et al., **329** , 873) This shows an asymmetric two-sided jet on the milliarcsecond scale.

$(1\text{mas} = 1.18 \text{ pc} = 3.65\times10^{18} \text{ cm})$

Features of jets from radio galaxies and quasars

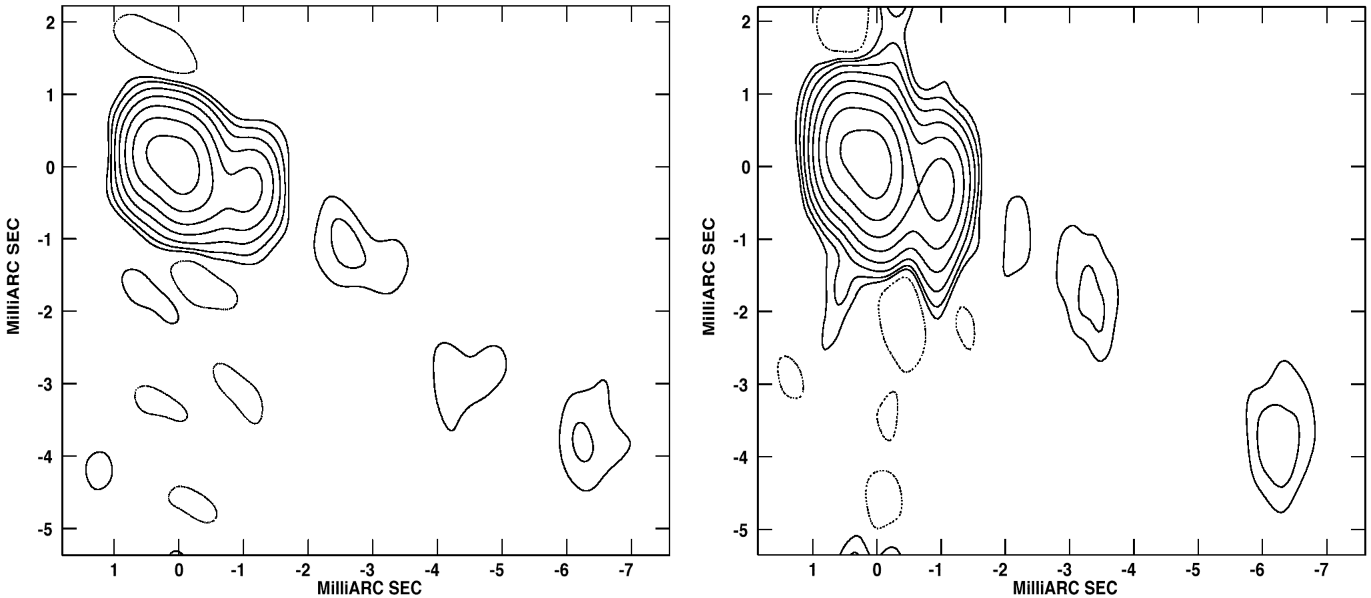
- We believe that the jets such as those from Cygnus A are ejected at relativistic speeds from a black hole in the nucleus.

- The knots in the jets are shock waves

- Two possibilities for creation of the shocks include variation in the flow velocity and the creation of disturbances in the supersonic flow via the Kelvin-Helmholtz instability

- In some quasars, in which we see one-sided jets, only, the knots move at apparent superluminal speeds.

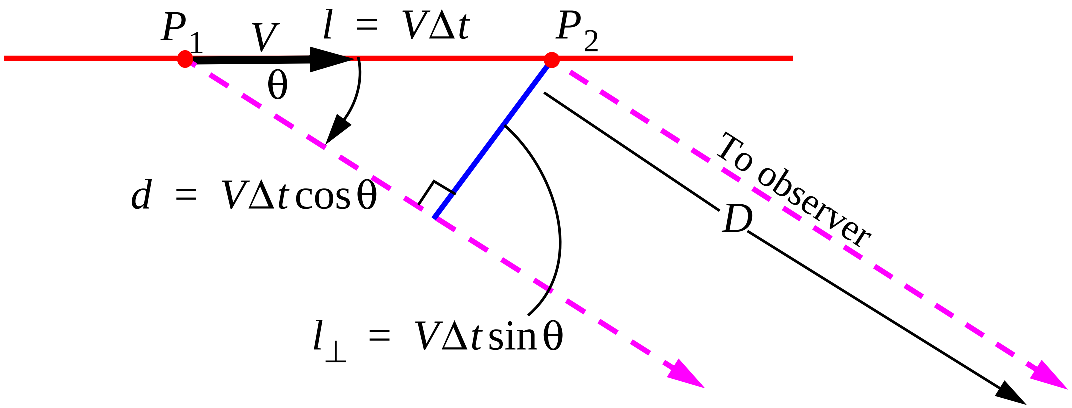
Superluminal motion in 3C273



Two images of the quasar 3C273 taken 9 days apart. Images like this are used to infer apparent superluminal motions of about $5c$. These 43 GHz images are from Mantovani et al. A&A, **354** , 497. Note that the jet is one-sided as a result of relativistic beaming.

2 Superluminal motion

2.1 Apparent transverse velocity



Consider an object which moves from P_1 to P_2 in a time Δt in the observer's frame. Let photons be emitted from P_1 and P_2 at times t_1 and t_2, respectively, with

$$\Delta t = t_2 - t_1$$

The times at which these photons are received by the observer are:

$$t_1^{\text{rec}} = t_1 + \frac{[D + V\Delta t\cos\theta]}{c}$$

$$t_2^{\text{rec}} = t_2 + \frac{D}{c}$$

$$\Rightarrow t_2^{\text{rec}} - t_1^{\text{rec}} = \Delta t - \frac{V\Delta t\cos\theta}{c} = \Delta t\left(1 - \frac{V}{c}\cos\theta\right)$$

The apparent distance moved by the object is

$$l_\perp = V\Delta t\sin\theta$$

Hence, the apparent velocity of the object is:

$$v_{\text{app}} = \frac{V\Delta t\sin\theta}{\Delta t\left(1 - \frac{V}{c}\cos\theta\right)} = \frac{V\sin\theta}{\left(1 - \frac{V}{c}\cos\theta\right)}$$

$$\frac{v_{\text{app}}}{c} = \frac{\frac{V}{c}\sin\theta}{\left(1 - \frac{V}{c}\cos\theta\right)}$$

In terms of

$$\beta_{app} = \frac{v_{app}}{c} \qquad \beta = \frac{V}{c}$$

$$\beta_{app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}$$

The non-relativistic limit is just $v_{app} = V\sin\theta$, as we would expect. However, note that this result is not a consequence of the Lorentz transformation, but a consequence of light travel time effects resulting from the finite speed of light.

2.2 Superluminal motion

For angles close to the line of sight, the effect of this equation can be dramatic. First, determine the angle for which the apparent velocity is a maximum:

$$\frac{d\beta_{app}}{d\theta} = \frac{(1-\beta\cos\theta)\beta\cos\theta - \beta\sin\theta\beta\sin\theta}{(1-\beta\cos\theta)^2}$$

$$= \frac{\beta\cos\theta - \beta^2}{(1-\beta\cos\theta)^2}$$

This derivative is zero when

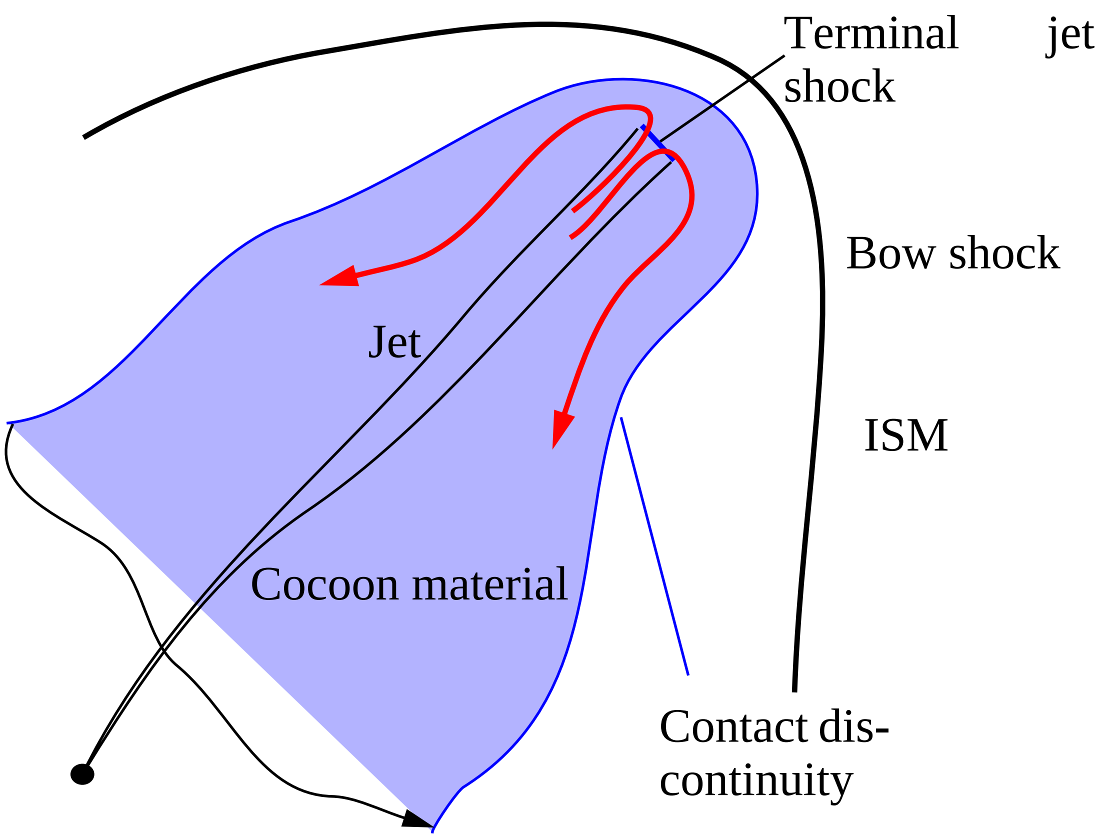
$$\cos\theta = \beta$$

and this implies that

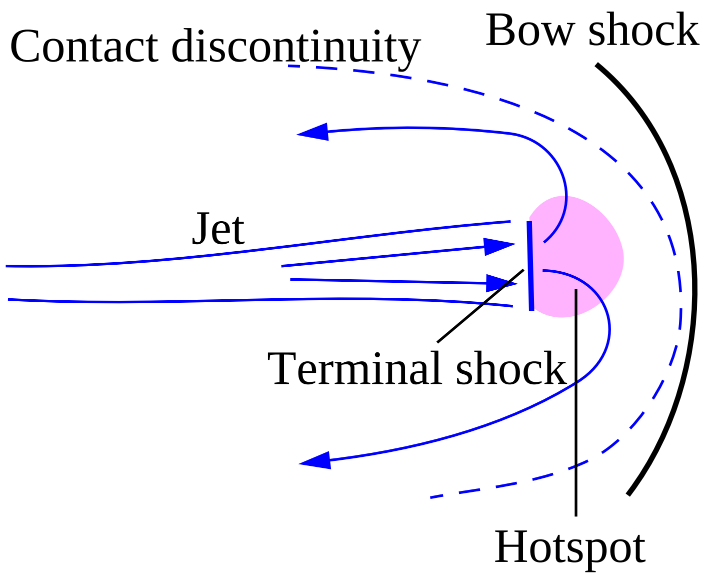
$$\beta_{app} = \frac{\beta\sin\theta}{1-\beta\cos\theta} = \frac{\beta\sqrt{1-\beta^2}}{1-\beta^2} = \frac{\beta}{\sqrt{1-\beta^2}} = \Gamma\beta$$

If $\Gamma \gg 1$ then $\beta \approx 1$ and the apparent velocity of an object can be larger than the speed of light.

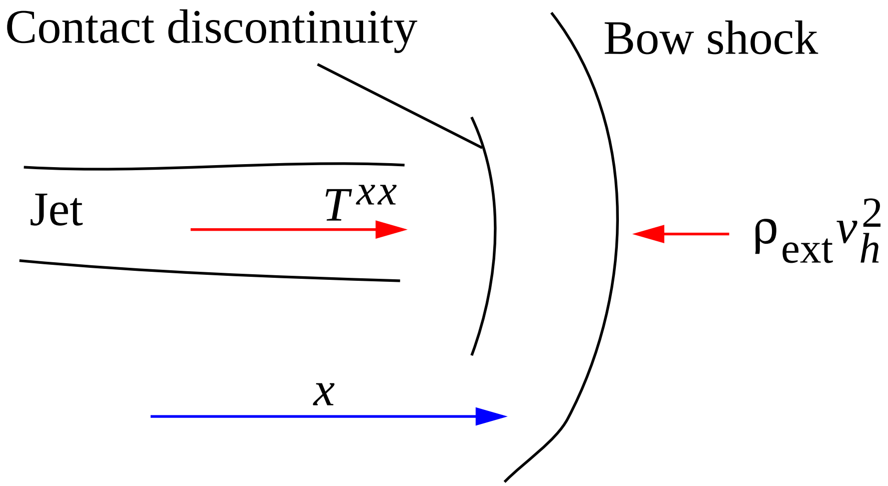
3 Speed of advance of a hotspot



This picture represents the generic situation of jet forcing it way through the interstellar medium. The speed of advance of the hotspot at the end of the jet is governed by the momentum balance at the end of the jet.



The speed of the hotspot is effectively governed by the thrust (force) of the jet against the interstellar medium through which it is travelling. The interaction region between jet and ISM is quite complex. However, we can analyse it approximately as follows:



The force balance in the frame of the contact

Let v_h be the speed of the hotspot in the lab frame.

We examine the balance between the force per unit area of the jet in this frame and the force per unit area of the incoming gas from the interstellar medium.

Balancing these two pressures,

$$T^{xx} = \rho_{\rm ext} v_h^2$$

where

$$T^{xx} = \Gamma^2 (e + p) \frac{v_x^2}{c^2} + p$$

Non-relativistic jet

It helps to gain some understanding of the physics if we first consider a non-relativistic, but supersonic jet. Then,

$$T^{xx} \approx \rho_{\rm jet} v_x^2 = \rho_{\rm ext} v_h^2$$

$$\Rightarrow v_h \approx \left(\frac{\rho_{\rm jet}}{\rho_{\rm ext}}\right)^{1/2} v_{\rm jet}$$

It is obvious from this expression, that when the jet is light compared to the external medium, as we believe all extragalactic jets to be, then the speed of advance of the hotspot is much less than the speed of the jet.

Relativistic jet

Let us assume that the jet consists of highly relativistic particles (for which there is good evidence) and that $\Gamma^2 \gg 1$. Then, $p = e/3$ and

$$T^{xx} \approx \Gamma^2 \times 4p \times \beta^2 \approx 4p\Gamma^2 = \rho_{\rm ext} v_h^2$$

The velocity of advance of the hotspot is then given by:

$$v_h \approx 2\left(\frac{p_{\text{jet}}}{\rho_{\text{ext}}}\right)^{1/2}\Gamma = 2\left(\frac{p_{\text{jet}}}{\mu n_{\text{ext}} m_p}\right)^{1/2}\Gamma$$

If the jet pressure is anything like the ambient ISM pressure (arguable but not definitely required) then

$$v_h \approx 2\left(\frac{p_{\text{ism}}}{\rho_{\text{ext}}}\right)^{1/2}\Gamma = 2\left(\frac{kT_{\text{ism}}}{\mu m_p}\right)^{1/2}\Gamma = 2a_s\Gamma$$

where a_s is the isothermal speed of sound in the ISM. Since we have assumed that $\Gamma \gg 1$, the motion of the hotspot in the ISM is supersonic and that is why there is a bow shock surrounding the waste material from the jet.

The hotspot pressure

Recall the relationship between pressure and pre-shock velocity for a shock

$$\beta_1 = \sqrt{\frac{1}{3}\left(\frac{3p_2 + p_1}{3p_1 + p_2}\right)}$$

It is also easy to derive

$$\Gamma^2 = \frac{1}{1-\beta_1^2} = \frac{9}{8} + \frac{3p_2}{8p_1}$$

$$\Rightarrow \frac{p_2}{p_1} = \frac{8}{3}\Gamma^2 - 3$$

Since the hotspot region is moving slowly at a subrelativistic speed, then we can consider the terminal jet shock to be at rest. Hence, the shock relations derived for the frame of the shock apply and

$$\frac{\text{Hotspot pressure}}{\text{Jet pressure}} = \frac{p_{\text{hs}}}{p_{\text{jet}}} = \frac{8}{3}\Gamma_{\text{jet}}^2 - 3$$

3.1 Application to Cygnus A

Estimates of the jet and hotspot pressures in Cygnus A give

$$p_{\text{jet}} \approx 3\times10^{-10}\ \text{dynes cm}^{-2} = 3\times10^{-10}\ \text{N m}^{-2}$$

$$p_{\text{hs}} \approx 3\times10^{-9}\ \text{dynes cm}^{-2} = 3\times10^{-10}\ \text{N m}^{-2}$$

This gives:

$$\frac{8}{3}\Gamma_{\text{jet}}^2 - 3 = 10 \Rightarrow \Gamma_{\text{jet}} \approx 2.2$$

The velocity of the hotspot is:

$$v_h \approx 2\left(\frac{p_{\text{jet}}}{\rho_{\text{ext}}}\right)^{1/2}\Gamma = 2\left(\frac{p_{\text{jet}}}{\mu n_{\text{ext}} m_p}\right)^{1/2}\Gamma$$

X-ray observations inform us that the medium surrounding Cygnus A has a temperature of about $10^7\,{}^\circ\mathrm{K}$ and a density~ $10^{-2}\ \mathrm{cm}^{-3}$ ($10^4\ \mathrm{m}^{-3}$). Hence the velocity of advance into the surrounding medium is

$$v_h \sim 0.025c$$

That is, the speed of advance of the hotspot is about 1/40 th of the velocity of the jet.