Project on Visualising the Kelvin-Helmholtz Instability

1 Introduction

The Kelvin-Helmholtz instability is an important instability in fluid dynamics. The first investigations were of the instability of two relatively moving fluids in plane parallel streams – so-called free shear flow. The book by Chandrasekhar (1961) constains a good exposition of the incompressible instability as well as references to some of the literature. I also have some lecture notes on the compressible instability which are useful. These are provided with these notes (Instabilities.pdf)

The importance of the surface instability relates to its role in the production of turbulence. Non-linear development leads to turbulent mixing in a "free shear layer".

In astrophysics the developing ideas related to jets in radio galaxies and quasars and the development of models led to renewed interest in this area and several lines were followed. Turland & Scheuer (1976) and Blandford & Pringle (1976) generalised the instability to relativistic shear layers. Several people have also considered the instability of axisymmetric jets, both non-relativistic and relativistic and one important concept here is the notion of body and reflecting modes. The former involve the perturbation of the jet as a whole; the second are the analogy of unstable waves found for shear layers and are mainly confined to the surface. Important references are Payne & Cohn (1985), Birkinshaw (1984), Birkinshaw (1991), Hardee (1987a), Hardee (1987b) and Bicknell & Begelman (1996) as well as references to some of the older literature in these papers. An important consequence of body modes is that they lead to gross deflections of a jet and thereby to the possible formation of shock waves in a supersonic flow.

2 The unstable modes

The unstable modes ar determined from the dispersion relation resulting from perturbations to the zeroth order flow. For an axisymmetric jet the perturbations to the jet density, velocity etc. are expressed in cylindrical coordinates (r, ϕ, z) in the form

$$\text{Perturbation} \propto f(r)\ \exp[i(\omega t + m\phi - k_z z)] \qquad (1)$$

where $f(r)$ is an appropriate radial function determined from the perturbation equations. Substitution of such equations into the fluid equations and utilising continuity conditions at the boundaries of the two fluids leads to a dispersion equation for ω/k_z. One can take k_z real and solve for ω. Unstable waves are identified by an imaginary part of ω corresponding to exponentially growing waves. This is known as a *temporal instability* analysis.

An approach which is suited to the growth of waves as a result of time dependent perturbations, is to take ω real and allow k_z to be complex. Values of k_z with an imaginary part correspond to spatially growing waves. This is often the approach of choice for astrophysical situations.

The instability modes of a jet depend upon the Mach number, M and density ratio $\eta = \rho_{\rm jet}/\rho {\rm ext}$ where $\rho_{\rm jet}$ and ρext are the jet and external densities.

3 The project

3.1 Components

This project will consist of the following:

- A derivation of the dispersion equation for a non-relativistic axisymmetric jet.
- Numerical solution of the dispersion equation for various modes of interest ($m = 0, 1, 2$ and body and reflecting modes).
- Numerical determination of the most rapidly growing modes. (See Bicknell & Begelman (1996) for an example of this for a relativistic jet.)
- The use of a visualisation package such as IDL or MATLAB to plot the pressure and velocity fields from various persepectives (e.g. r, z, r, ϕ at constant z resulting from the most rapidly growing modes.
- Visualisation of the shape of the jet for the different modes. This could take the form of sectional plots or 3D plots of the deformed surface of the jet.

The last two parts are very useful components of this exercise since they give us an idea of how the non-linear development of the KH instability may proceed. Note that the dispersion relations and the pressure and velocity fields, depend upon various orders of Bessel functions, so that whatever numerical package you use to do the analysis, these special functions would need to be provided.

I shall leave it to your own judgement and artistic expertise, just how you carry out the visualisation.

3.2 Parameters

Since there may be more than one project involved, here are the parameters for each of the two projects.

	Project 1 $M = 1, 10$	Project 2 $M = 0.5, 5$
$\eta = 0.01$	$m = 0, 1, 2$	$m = 0, 1, 2$
$\eta = 1$	$m = 0, 1, 2$	$m = 0, 1, 2$

It may be a tall order to carry out all of the above both for reflecting and body modes. The mode I would most like you to concentrate on is the $m = 1$ mode and to carry the project through to the extent that you can visualise at least the pressure field for this mode.

4 Obtaining references

Note that the astronomical journal references may be obtained from the MSO library but also via the web through the NASA Astrophysical Data System abstract service: `http://adsabs.harvard.edu/abstract_service.html`

References

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