Statistical Analysis of Solar and Stellar Properties

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Abstract—We review the results of Robles et al. [1] in which a simultaneous multi-parameter comparison of solar and stellar properties and environments is presented. This 11-parameter analysis quantifies the (a)typicality of the Sun: we obtain a reduced solar $\chi^2_{\odot}/11 = 0.76 \pm 0.09$ and a probability of $29\% \pm 11\%$ that a star selected at random has a lower χ^2 value than solar. These two values are consistent with the idea that the Sun is a star selected at random rather than a special star. We also discuss the dependence of the results on different parameter and dataset selection criteria.

I. INTRODUCTION

Whether the Sun is a star selected at random from the bag of all stars or the Sun is a special star, may be connected with the evolution of life on Earth. If there is a life-enhancing, special stellar property, we could expect two things: i) the Sun has it because it hosts a life-bearing planet; ii) if such a property is special, a large fraction of the stars will not have it, and the Sun will stand out when compared to its peers ([2], [3]).

The (a)typicality of the Sun has been previously investigated by various studies (e.g. [4], [5], [6], [7]). Gustafsson's analysis concluded that the Sun is a normal star although it departs in properties like mass and amplitude of micro-variability ([4], [7]). Gonzalez's analysis on the other hand, suggests that the apparently anomalous solar parameters are clues about the habitability of the Earth ([5], [6]). These previous analyses have a similar methodology — an individual comparison of the Sun and the stars for a number of stellar properties, then each individual comparison is discussed and a qualitative overall result is proposed.

In 1998, Gustafsson [4] discussed the atypically large solar mass, and proposed an anthropic explanation — the Sun's high mass is probably related to our own existence. He suggested that the solar mass could hardly have been greater than $\sim 1.3 \, M_{\odot}$ since the main sequence lifetime of a $1.3 \, M_{\odot}$ star is ~ 5 billion years ([8]). He also discussed how the dependence of the width of the circumstellar habitable zone on the host star's mass probably favours host stars within the mass range $0.8{-}1.3 \, M_{\odot}$.

In 2008, Gustafsson [7] addressed the question of the Sun's uniqueness in his paper entitled "Is the Sun unique as a star — and if so, why?" After discussing the Sun's mass, chemical composition, activity, variability and the fact that the Sun is not a member of a binary system, he concludes that in answer to the question in his title, a reasonably

simple working hypothesis is that "The Sun is odd in certain respects since a habitable planetary system has to be there too." This working hypothesis assumes without support that habitable planetary systems are odd. Current exoplanet data are not of sufficient quality to address this question with much confidence. The exoplanet data are beginning to address the question of whether Jupiter-like planets are common ([9], [10]), but it is still the case that if the Sun were amongst the nearby Doppler target stars, Jupiter would be at the limits of detectability.

Gustafson's working hypothesis is not supported by our more quantitative result that the properties of the Sun are consistent with the Sun being a random star. Gustafson mentions that Robles et al. ([1], henceforth referred as R08) do not explicitly include micro-variability and binarity in our analysis. However, we did include binarity in one version of our analysis and found it to have no significant effect on our main conclusions (R08, p. 702). Similarly, the preliminary data on micro-variability ([11], [12]) indicate that the Sun's "low microvariability" is quantitatively ~ 1 sigma low — a result which would not affect our joint χ^2 analysis and our main result that the Sun appears to be a random star.

In order to quantify the degree of (a)typicality of the Sun, all the individual comparisons must be evaluated together in a multi-parameter analysis. Here we expand on the statistical analysis presented in R08.

The paper is organised as follows: Section II briefly discuses the parameters and samples to which the solar values are compared. Section III presents the calculation of the solar χ^2 . Section IV presents the simulation of stellar χ^2 values and the estimated probability of selecting a star with lower χ^2 than solar. Section V describes the advantages of our statistical analysis as well as our selection criteria for parameters that are correlated. Section VI discusses the dependence of the results on the selection of different parameters and different datasets for a given parameter.

II. PARAMETERS AND SAMPLES

The first part of the analysis is to compare the sun to other stars for a number of properties. Ideally, we would compare the Sun to a large, unbiased sample of stars for as many properties as desired. However, the construction of such a sample is impeded by observational limitations. Different surveys focus on different parameters and a survey's selection criteria can introduce significant biases in the properties of their sample. We have accounted for these biases in our analysis, which involves several stellar samples from the literature. Our selected samples are:

- Representative: for each property, the stellar distribution compared to the Sun is the least-biased possible with respect to that property.
- Independent: the selected properties must be largely independent of each other. Because correlations between stellar properties exist, we select maximally independent sets of data or subsets of data.
- Connected to habitability: The selected properties for the comparison are properties with a plausible connection to habitability.

The following are the 11 stellar (or environmental) properties we compare the Sun to: (1) mass, (2) age, (3) metallicity¹ [Fe/H], (4) carbon-to-oxygen ratio [C/O], (5) magnesium-tosilicon ratio [Mg/Si], (6) rotational velocity (spin, not orbital), (7) eccentricity of the star's galactic orbit e, (8) maximum height to which the star rises above the galactic plane Z_{max} , (9) mean galactocentric radius R_{Gal} , (10) the mass of the star's host galaxy M_{gal} , (11) the mass of the star's host group of galaxies M_{group} . Table I contains the distribution ranges and medians for every parameter considered in the analysis as well as the adopted solar values.

 TABLE I

 PROPERTIES OF PARAMETERS' DISTRIBUTIONS

Property	Range	Median	Solar
		$\mu_{1/2}$	Value
Mass [M _☉]	0.08 - 2	0.33	$\equiv 1$
Age [Gyr]	0 - 15	5.4	$4.9^{+3.1}_{-2.7}$
[Fe/H]	-1.20 - +0.46	-0.08	$\equiv 0$
[C/O]	-0.22 - +0.32	0.07	$\equiv 0$
[Mg/Si]	-0.18 - +0.14	0.01	$\equiv 0$
$v \sin i [\text{km s}^{-1}]$	0 - 36	2.51	1.28
e	0 - 1	0.10	0.036 ± 0.002
$Z_{\rm max}$ [kpc]	0 - 9.60	0.14	0.104 ± 0.006
R _{Gal} [kpc]	0 - 30	4.9	7.62 ± 0.32
M _{gal} [M _☉]	$10^7 - 10^{12}$	$10^{10.2}$	$10^{10.55\pm0.16}$
M _{group} [M _☉]	$10^9 - 10^{13}$	$10^{11.1}$	$10^{10.91\pm0.07}$

The details of the parameter selection as well as the construction of each individual stellar sample can be found in Section 2 and Table 1 of R08.

III. Solar χ^2_\odot Estimates

A. Simple Solar χ^2 -analysis

Figure 1 shows the comparison of the Sun to the 11 properties' distributions. The Sun, indicated by the yellow " \odot " symbols, scatters around the distributions medians (black filled circles). The dark and light green shades around the medians represent the 68% and 95% zones of each distribution (adapted from [13]). We would like to know if these solar properties, taken as a group, are consistent with noise, i.e., is the pattern of the Sun in Figure 1 consistent with the values of a star selected at random from the stellar distributions? We take a

¹Metallicity: [Fe/H] is the fractional abundance of Fe relative to hydrogen, compared to the same ratio in the Sun: $[Fe/H] \equiv \log(Fe/H)_{\star} - \log(Fe/H)_{\odot}$.

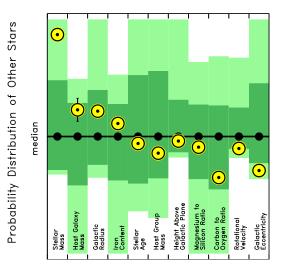


Fig. 1. Solar values of 11 properties compared to the distribution for each property (adapted from [13]). Each distribution's median value is indicated by a small filled black circle. The dark and light green shades around the medians represent the 68% and 95% zones respectively. The Sun is indicated by the yellow " \odot " symbols.

 χ^2 approach to answering this question. First we estimate the solar χ^2_{\odot} by adding contributions from each of the 11 properties. We find:

$$\chi_{\odot}^{2} = \sum_{i=1}^{N=11} \frac{(x_{\odot,i} - \mu_{1/2,i})^{2}}{\sigma_{68,i}^{2}} = 7.88_{-0.30}^{+0.08} \tag{1}$$

where *i* is the property index, N = 11 is the number of properties we are considering, $\mu_{1/2,i}$ is the median of the i^{th} stellar distribution and $\sigma_{68,i}$ is the difference between the median and the upper or lower 68% zone, depending on whether the solar value $x_{\odot,i}$ is above or below the median. The uncertainty on χ^2_{\odot} is obtained using the uncertainties of $x_{\odot,i}$.

With 11 degrees of freedom, the reduced χ^2_{\odot} is $\chi^2_{\odot} / 11 = 0.72^{+0.01}_{-0.03}$. Since $\chi^2_{\odot} / 11 < 1$, the Sun's properties are consistent with the Sun being a randomly selected star. We find the probability of finding a star with a χ^2_{\star} value lower than the solar χ^2_{\odot} , for N = 11 degrees of freedom in the standard way [14] and obtain:

$$P(<\chi_{\odot}^{2} = 7.88^{+0.08}_{-0.30}|11) = 0.28^{+0.01}_{-0.03}$$
(2)

If this value were close to 1, almost all other stars would have lower χ^2 values and we would have good reason to suspect that the Sun is not a typical star. However, this preliminary low value of 0.28 indicates that if a star is chosen at random, the probability that it will be more typical (~ have a lower χ^2 value) than the Sun (with respect to the eleven properties analysed here), is only $28^{+1}_{-3}\%$. We conclude that the Sun is a typical star. In the following, we improve on this preliminary analysis but come to a similar conclusion.

B. Improved Estimate of χ^2_{\odot}

Equation (1) can be improved upon by taking into account: *a*) the non-Gaussian shapes of the stellar distributions and *b*)

the larger uncertainties of the medians of smaller samples (our smallest sample is ~ 100 stars). These two improvements, as well as the independence between properties are discussed in Section V.

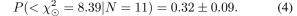
We employ a bootstrap analysis [15] to randomly resample data (with replacement) and derive a more accurate estimate of χ^2_{\odot} .

For every iteration, each parameter's stellar distribution is randomly resampled and a χ^2_{\odot} value is calculated using Eq. (1). The uncertainties $\sigma_{\odot,i}$ of the solar values $x_{\odot,i}$ are also included in the bootstrap method: for every iteration, the solar value for each parameter is replaced in Eq. (1) by a randomly selected value from a normal distribution with median $\mu_{1/2,i} = x_{\odot,i}$ and standard deviation $\sigma_{\odot,i}$. The process was iterated 100,000 times, although the resulting distribution varies very little once the number of iterations reaches ~ 10,000. The median of this distribution and the error on the median yields our improved value for the reduced χ^2_{\odot} (Fig. 2).

We obtain $\chi^2_{\odot} = 8.39 \pm 0.96$. Figure 2 shows the resulting solar χ^2 distribution. The median of this distribution is our adopted solar χ^2_{\odot} value. Dividing our adopted solar χ^2 by the number of degrees of freedom (N = 11) gives our adopted reduced solar χ^2 value:

$$\chi_{\odot}^2 / 11 = 0.76 \pm 0.09 \tag{3}$$

The standard conversion of this into a probability of finding a star with a lower χ^2 value than χ^2_{\odot} (assuming normally distributed independent variables) yields:



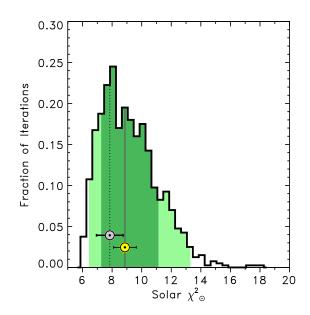


Fig. 2. Bootstrapped solar χ^2 distribution. The median of the distribution (yellow " \odot ") is $\chi^2_{\odot} = 8.39 \pm 0.96$. This should be compared to the solar χ^2_{\odot} value from Eq. 1: $7.88^{+0.08}_{-0.30}$ which is over-plotted (grey " \odot " on dotted line). The dark and light green shades around the median represent the 68% and 95% zones respectively.

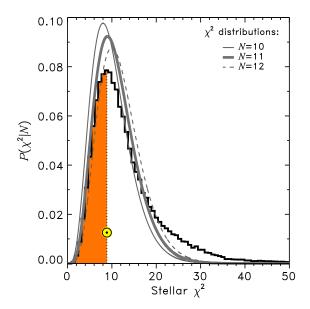


Fig. 3. Stellar χ^2_{\star} distribution from our Monte Carlo simulation. $P_{\rm MC}(\leq \chi^2_\odot = 8.39) = 0.29 \pm 0.11$ (represented by the orange shade) is calculated integrating from $\chi^2 = 0$ to $\chi^2 = \chi^2_\odot$. For comparison, three χ^2 distribution-curves are over-plotted with 10, 11, and 12 degrees of freedom. The standard probability from the N = 11 curve yields: $P(<\chi^2_\odot = 8.39|N = 11) = 0.32 \pm 0.09$. The longer tail of the Monte Carlo distribution is produced by the longer super-Gaussian tails of the stellar distributions.

IV. Stellar χ^2_{\star} Probability Estimates

To quantify how typical the Sun is with respect to our 11 properties, we compare the solar $\chi^2_{\odot}(=8.39)$ to the distribution of χ^2 values obtained from the other stars in the samples.

We perform a *Monte Carlo* simulation [16] to calculate an estimate of each star's χ^2 value (" χ^2_{\star} "). For every iteration, we randomly select a star from each stellar distribution. We then calculate its χ^2_{\star} value by replacing the solar value $x_{\odot,i}$ with that star's value $x_{\star,i}$ in Eq. (1). This process was repeated 100,000 times to create our Monte Carlo stellar χ^2 distribution. The histogram shown in Figure 3 is the resulting montecarlo stellar χ^2 distribution. Three standard χ^2 distributions have been over-plotted for comparison (N = 10, 11, 12). The probability of finding a star with χ^2 lower than or equal to solar is:

$$P_{\rm MC}(\leq \chi_{\odot}^2 = 8.39 | N = 11) = 0.29 \pm 0.11$$
 (5)

The Monte Carlo χ^2 distribution has a similar shape to the standard χ^2 distribution function for N = 11, and thus both yield similar probabilities: $P_{\rm MC}(\leq \chi^2) = 0.29 \sim P(\leq \chi^2) = 0.32$ (Eqs. 4 and 5).

V. BOOTSTRAP ADVANTAGES AND PARAMETER CORRELATIONS

In Section III-B and Section IV we performed two different bootstrap analyses to improve our estimates of χ^2_{\odot} and $P_{\rm MC} (\leq \chi^2_{\odot})$. The bootstrap analysis addresses two issues overlooked by a simple χ^2 analysis and probability estimate (Eqs. 1 & 2):

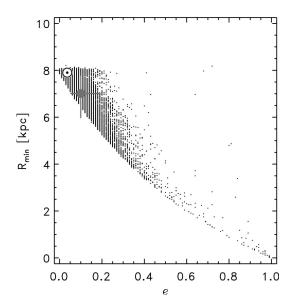


Fig. 4. Correlation between e and R_{\min} for 13240 A5–K2 stars from Nordström et al. [17].

The (non-)Gaussianity of the individual property distributions and, the errors associated with small-number statistics.

a) non-Gaussianity: Because the bootstrap is a nonparametric method, the distributions need not be Gaussian: the longer tails in the simulated distributions in Figures 2 and 3 account for the non-Gaussianity of the properties' distributions.

b) small-number statistics: The larger error contributed by distributions with a small number of stars is accounted for by the random resampling of the original distributions, i.e., the median values are less certain for smaller samples and this uncertainty is included in our improved estimate of χ^2_{\odot} and its uncertainty. The uncertainty of the median of each re-sampled distribution varies inversely proportionally to the square root of the number of stars in the distribution: $\Delta \mu_{1/2,i} \propto 1/\sqrt{N_{\star,i}}$.

Special attention should be given to an effect not accounted for by our solar χ^2 bootstrap analysis: correlations between parameters. Correlations between parameters reduce the effective number of degrees of freedom (i.e. $N_{\rm effective} < 11$). Without correction, this results in an underestimation of the reduced χ^2_{\odot} value, thus, making the Sun appear more typical than it is.

To minimise this correlations effect, we selected maximally independent parameters. For example, Figures 4 and 5 show the correlation plots between the Galactic eccentricity e, the minimum galactocentric radius R_{\min} and the height above the Galactic plane Z_{\max} . R_{\min} is highly correlated with e (Fig. 4), so only one of those properties should be included in the analysis. When comparing the correlations of Z_{\max} with R_{\min} and Z_{\max} with e (top and bottom panels in Fig. 5 respectively), it can be seen that e is less correlated with Z_{\max} than R_{\min} . Thus, e and Z_{\max} were included in the analysis.

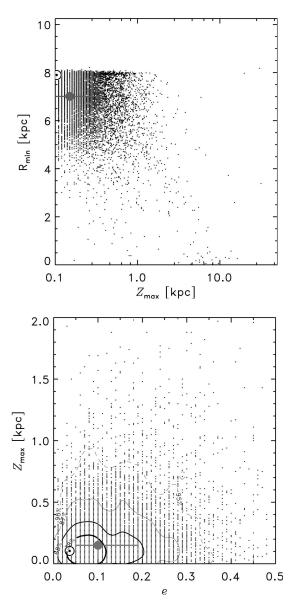
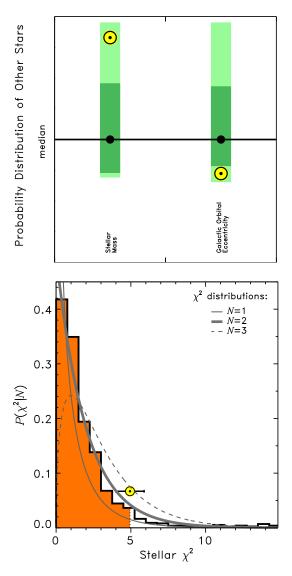


Fig. 5. Top: Z_{max} versus R_{min} . Bottom: *e* versus Z_{max} , the contour lines in decreasing thickness encompass: 38%, 68%, 87% and 95% of the stars. *e* is less correlated with Z_{max} than R_{min} . Data same as Figure 4.

VI. WHAT IF YOU CHOOSE DIFFERENT PARAMETERS?

In Section III and IV we made improved estimates of the χ^2_{\odot} and $P(\leq \chi^2_{\odot})$ for the selected 11 properties distributions. But how robust is this result? The probability of finding a star with χ^2_{\star} lower than or equal to χ^2_{\odot} , depends on the properties selected for the analysis. For example, what if we choose only mass and eccentricity? In that case the analysis yields a reduced $\chi^2_{\odot}/2 = 2.47 \pm 0.49$ per degree of freedom, and a probability $P_{\rm MC}(\chi^2_{\star} \leq \chi^2_{\odot}) = 0.92 \pm 0.05$. The top panel in Figure 6 shows the selection of stellar Mass and stellar galactic eccentricity. The bottom panel shows the simulated stellar χ^2 distribution obtained with two degrees of freedom. In this case the Sun would appear mildly (~ 2σ) anomalous. However, the selection of only these two parameters is not random — we



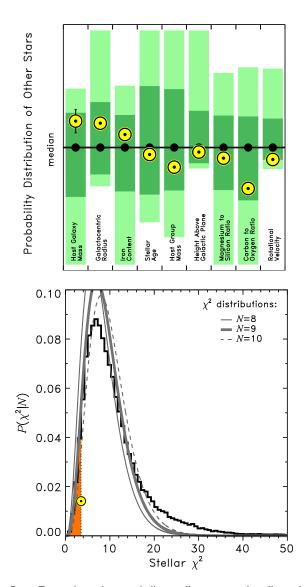


Fig. 6. Top: solar values of stellar mass and stellar galactic eccentricity. Bottom: Stellar χ^2_{\star} distribution from our Monte Carlo simulation. If only mass and eccentricity are selected, the obtained $P_{\rm MC}(\leq \chi^2_{\odot}) = 0.92 \pm 0.05$. For comparison, three χ^2 distribution-curves are over-plotted with 1, 2 and 3 degrees of freedom.

Fig. 7. Top: solar values excluding stellar mass and stellar galactic eccentricity from the analysis. Bottom: Stellar χ^2_{\star} distribution from our Monte Carlo simulation. If mass and eccentricity are excluded, the obtained $P_{\rm MC} (\leq \chi^2_{\odot}) = 0.07 \pm 0.04$. For comparison, three χ^2 distribution-curves are over-plotted with 8, 9 and 10 degrees of freedom.

know a priori that these are the most anomalous parameters available.

If on the other hand, we chose to remove mass and eccentricity from the analysis, the analysis yields a reduced $\chi^2_{\odot}/9 = 0.39 \pm 0.09$ per degree of freedom, and a probability $P_{\rm MC}(\chi^2_{\star} \leq \chi^2_{\odot}) = 0.07 \pm 0.04$, which is anomalously low, i.e., the Sun's values are improbably close to the distributions' medians. Mass and eccentricity need to be included to prevent the Sun from being improbably typical (Fig. 7).

The presented results are robust to the use of different datasets (by different studies) for any given parameter. e.g. replacing the stellar metallicity dataset compiled by Grether & Lineweaver [18] with the metallicity dataset compiled by Favata et al. [19]. This independence is valid for the 8

parameters for which we have more than one set of data: when using these different sets of data, the obtained χ^2_{\odot} and $P_{MC}(\leq \chi^2)$ values are consistent with the values obtained with our final sets selection ($\chi^2_{\odot}/11 = 0.76 \pm 0.09$ and $P_{MC}(\leq \chi^2) = 0.29 \pm 0.11$).

VII. RESULTS

Our simple $\chi^2_{\odot} = 7.88$ estimate increased to 8.39 and the uncertainty increased by a factor of ~ 3 after non-Gaussian and small-number-statistics effects were included as additional sources of uncertainty. Our improved analysis yields $P_{\rm MC} (\leq \chi^2_{\odot})$, with a longer tail and brings the probability down from 0.32 ± 0.09 to 0.29 ± 0.11 .

Our analysis for the solar χ^2_{\odot} values and the probabilities $P(\langle \chi^2_{\odot})$ can be summarised as follows:

- 1) A simple solar χ^2_{\odot} analysis (Eq. 1) increases from $\chi^2_{\odot} = 7.88^{+0.08}_{-0.30}$ to $\chi^2_{\odot} = 8.39 \pm 0.96$ with our solar χ^2 bootstrap.
- χ bootstrap.
 2) Accordingly, the reduced simple solar χ²_☉ for 11 degrees of freedom, varies from χ²_☉/11 = 0.72^{+0.01}_{-0.03} to χ²_☉/11 = 0.76 ± 0.09 (Eq. 3) with our solar χ² bootstrap.
 3) Using the simple and improved χ²_☉ values, the probabilities P that in a χ² distribution with 11 degrees of freedom, a random star has a χ²_⋆ ≤ χ²_☉ are P(χ²_⋆ ≤ 7.88|11) = 0.28^{+0.01}_{-0.03} (Eq. 2) and P(χ²_⋆ ≤ 8.39|11) = 0.32 ± 0.09 (Eq. 4) 0.32 ± 0.09 (Eq. 4).
- 4) Using our Monte Carlo stellar χ^2_{\star} simulation, we estimate a $P_{\rm MC}(\leq 8.39|11) = 0.29 \pm 0.11$. This probability is 3% smaller than the one obtained from Eq. 4.

VIII. CONCLUSION

When the Sun is compared to other stars, simultaneously in mass, age, metallicity, carbon-to-oxygen ratio, magnesium-tosilicon ratio, rotational velocity, galactic eccentricity, height above the Galactic plane, galactocentric radius, host galaxy mass and host group mass, we find that it is very typical. Our bootstrap analysis yields a reduced solar $\chi^2_{\odot}/11 = 0.76 \pm 0.09$ and a probability of just $29\% \pm 11\%$ that a star selected at random has a lower χ^2_{\star} value than the Sun — somewhat closer to the distribution medians than random values are expected to be. Comparable figures are obtained by a simplistic χ^2 and probability analysis.

The 11 parameters chosen represent the set of largely independent well-observed properties that are plausibly related to habitability. There is no indication that the Sun has any special stellar properties that were required for the development of life in the solar system.

When particular subsets of our 11 parameters are considered, e.g. only mass and eccentricity, the Sun appears anomalous. However, the posterior selection of such subsets precludes any conclusions that may follow, about the (a)typicality of the Sun or the importance of such properties for life.

For 8 parameters, more than one dataset was available and our results were consistent when the analysis was performed using these different datasets for a given parameter(s).

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