

RESEARCH ARTICLES

- 66. The mean equatorial radius was derived from the harmonic model (65) based on a 1° sampling of an equatorial profile. This value is 200 m larger than was estimated from earlier data (5) but is within the error estimate of the earlier value. The uncertainty corresponds to the standard error of the mean of the 360 equatorial samples.
- 67. We acknowledge the MOLA instrument team and the MGS spacecraft and operation teams at the Jet Propulsion Laboratory and Lockheed-Martin Astronau-

tics for providing the engineering foundation that enabled this analysis. We also thank G. Elman, P. Jester, and J. Schott for assistance in altimetry processing, D. Rowlands and S. Fricke for help with orbit determination, S. Zhong for assistance with the Hellas relaxation calculation, and G. McGill for a constructive review. The MOLA investigation is supported by the NASA Mars Global Surveyor Project.

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such a young age by such a wide array of recent independent measurements.

Method

Any measurement of a function of h , Ω_m , and Ω_Λ can be included in a joint likelihood

$$L(h, \Omega_m, \Omega_\Lambda) = \prod_{i=1}^N L_i \quad (1)$$

which I take as the product of seven of the most recent independent cosmological constraints (Table 1 and Fig. 3). For example, one of the L_i in Eq. 1 represents the constraints on h . Recent measurements can be summarized as $\bar{h} = 0.68 \pm 0.10$ (16). I represent these measurements in Eq. 1 by the likelihood

$$L_{\text{Hubble}}(h) = \exp \left[-0.5 \left(\frac{h - \bar{h}}{0.10} \right)^2 \right] \quad (2)$$

Another L_i in Eq. 1 comes from measurements of the fraction of normal baryonic matter in clusters of galaxies (14) and estimates of the density of normal baryonic matter in the universe [$\Omega_b h^2 = 0.015 \pm 0.005$ (15, 18)]. When combined, these measurements yield $\bar{\Omega}_m h^{2/3} = 0.19 \pm 0.12$ (19), which contributes to the likelihood through

$$L_{\text{baryons}}(h, \Omega_m) = \exp \left[-0.5 \left(\frac{\Omega_m h^{2/3} - \bar{\Omega}_m h^{2/3}}{0.12} \right)^2 \right] \quad (3)$$

The $(\Omega_m, \Omega_\Lambda)$ -dependencies of the remaining five constraints are plotted in Fig. 3 (20). The 68% confidence level regions derived from CMB and SNe (Fig. 3, A and B) are nearly orthogonal, and the region of overlap is relatively small. Similar complementarity exists between the CMB and the other data sets (Fig. 3, C through E). The combination of them all (Fig. 3F) yields $\Omega_\Lambda = 0.65 \pm 0.13$ and $\Omega_m = 0.23 \pm 0.08$ (21).

This complementarity is even more important (but more difficult to visualize) in three-dimensional parameter space, $(h, \Omega_m, \Omega_\Lambda)$. Although the CMB alone cannot tightly constrain any of these parameters, it does have a strong preference in the three-dimensional space $(h, \Omega_m, \Omega_\Lambda)$. In Eq. 1, I used $L_{\text{CMB}}(h, \Omega_m, \Omega_\Lambda)$, which is a generalization

A Younger Age for the Universe

Charles H. Lineweaver

The age of the universe in the Big Bang model can be calculated from three parameters: Hubble's constant, h ; the mass density of the universe, Ω_m ; and the cosmological constant, Ω_Λ . Recent observations of the cosmic microwave background and six other cosmological measurements reduce the uncertainty in these three parameters, yielding an age for the universe of 13.4 ± 1.6 billion years, which is a billion years younger than other recent age estimates. A different standard Big Bang model, which includes cold dark matter with a cosmological constant, provides a consistent and absolutely time-calibrated evolutionary sequence for the universe.

In the Big Bang model, the age of the universe, t_o , is a function of three parameters: h , Ω_m , and Ω_Λ (1). The dimensionless Hubble constant, \bar{h} , tells us how fast the universe is expanding. The density of matter in the universe, Ω_m , slows the expansion, and the cosmological constant, Ω_Λ , speeds up the expansion (Fig. 1).

Until recently, large uncertainties in the measurements of h , Ω_m , and Ω_Λ made efforts to determine t_o ($h, \Omega_m, \Omega_\Lambda$) unreliable. Theoretical preferences were, and still are, often used to remedy these observational uncertainties. One assumed the standard model ($\Omega_m = 1, \Omega_\Lambda = 0$), dating the age of the universe to $t_o = 6.52/h$ billion years old (Ga). However, for large or even moderate h estimates (≥ 0.65), these simplifying assumptions resulted in an age crisis in which the universe was younger than our Galaxy ($t_o \approx 10 \text{ Ga} < t_{\text{Gal}} \approx 12 \text{ Ga}$). These assumptions also resulted in a baryon crisis in which estimates of the amount of normal (baryonic) matter in the universe were in conflict (2, 3).

Evidence in favor of $\Omega_m < 1$ has become more compelling (4–8), but Ω_Λ is still often assumed to be zero, not because it is measured to be so, but because models are simpler without it. Recent evidence from supernovae (SNe) (4, 5) indicates that $\Omega_\Lambda > 0$. These SNe data and other data exclude the standard Einstein-deSitter model ($\Omega_m = 1, \Omega_\Lambda = 0$). The cosmic microwave background (CMB), on the other hand, excludes models with low Ω_m and $\Omega_\Lambda = 0$ (3). With both high and low Ω_m excluded, Ω_Λ cannot be zero. Combining CMB measure-

ments with SNe and other data, I (9) have reported $\Omega_\Lambda = 0.62 \pm 0.16$ [see (10–12) for similar results]. If $\Omega_\Lambda \neq 0$, then estimates of the age of the universe in Big Bang models must include Ω_Λ . Thus, one must use the most general form: $t_o = f(\Omega_m, \Omega_\Lambda)/h$ (13).

Here, I have combined recent independent measurements of CMB anisotropies (9), type Ia SNe (4, 5), cluster mass-to-light ratios (6), cluster abundance evolution (7), cluster baryonic fractions (14), deuterium-to-hydrogen ratios in quasar spectra (15), double-lobed radio sources (8), and the Hubble constant (16) to determine the age of the universe. The big picture from the analysis done here is as follows (Figs. 1 and 2): The Big Bang occurred at ~ 13.4 Ga. About 1.2 billion years (Gy) later, the halo of our Galaxy (and presumably the halo of other galaxies) formed. About 3.5 Gy later, the disk of our Galaxy (and presumably the disks of other spiral galaxies) formed. This picture agrees with what we know about galaxy formation. Even the recent indications of the existence of old galaxies at high redshift (17) fit into the time framework determined here. In this sense, the result is not surprising. What is new is the support given to

Table 1. Parameter estimates from non-CMB measurements. I refer to these as constraints. I use the error bars cited here as 1σ errors in the likelihood analysis. The first four constraints are plotted in Fig. 3, B through E.

Method	Reference	Estimate
SNe	(35)	$\Omega_m^{\Lambda=0} = -0.28 \pm 0.16, \Omega_m^{\text{flat}} = 0.27 \pm 0.14$
Cluster mass-to-light	(6)	$\Omega_m^{\Lambda=0} = 0.19 \pm 0.14$
Cluster abundance evolution	(7)	$\Omega_m^{\Lambda=0} = 0.17^{+0.28}_{-0.10}, \Omega_m^{\text{flat}} = 0.22^{+0.25}_{-0.10}$
Double radio sources	(8)	$\Omega_m^{\Lambda=0} = -0.25^{+0.70}_{-0.50}, \Omega_m^{\text{flat}} = 0.1^{+0.50}_{-0.20}$
Baryons	(19)	$\Omega_m h^{2/3} = 0.19 \pm 0.12$
Hubble	(16)	$h = 0.68 \pm 0.10$

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of $L_{\text{CMB}}(\Omega_m, \Omega_\Lambda)$ (Fig. 3A) (22). To convert the three-dimensional likelihood $L(h, \Omega_m, \Omega_\Lambda)$ of Eq. 1 into an estimate of the age of the universe and into a more easily visualized two-dimensional likelihood, $L(h, t_o)$, I computed the dynamic age corresponding to each point in the three-dimensional space $(h, \Omega_m, \Omega_\Lambda)$. For a given h and t_o , I then set $L(h, t_o)$ equal to the maximum value of $L(h, \Omega_m, \Omega_\Lambda)$

$$L(h, t_o) = \max[L(h, \Omega_m, \Omega_\Lambda) |_{t(h, \Omega_m, \Omega_\Lambda) = t_o}] \quad (4)$$

This has the advantage of explicitly displaying the h dependence of the t_o result. The joint likelihood $L(h, t_o)$ of Eq. 4 yields an age for the universe of $t_o = 13.4 \pm 1.6$ Ga (Fig. 4). This result is a billion years younger than other recent age estimates.

What one uses for $L_{\text{Hubble}}(h)$ in Eq. 1 is particularly important because, in general, we expect the higher h values to yield younger ages. Table 2 contains results from a variety of h estimates, assuming various central values and various uncertainties around these values. The main result of $t_o = 13.4 \pm 1.6$ Ga has used

$h = 0.68 \pm 0.10$ but does not depend strongly on the central value assumed for Hubble's constant (as long as this central value is in the most accepted range, $0.64 \leq h \leq 0.72$) or on the uncertainty of h (unless this uncertainty is taken to be very small). Assuming an uncertainty of 0.10, age estimates from using $h = 0.64, 0.68,$ and 0.72 are 13.5, 13.4, and 13.3 Ga, respectively (Fig. 2). Using a larger uncertainty of 0.15 with the same h values does not substantially change the results, which are 13.4, 13.3, and 13.2 Ga, respectively. For both groups, the age difference is only 0.2 Gy. If $t_o \propto 1/h$ were adhered to, this age difference would be 1.6 Gy. Outside the most accepted range, the h dependence becomes stronger and approaches $t_o \propto 1/h$ (23).

To show how each constraint contributes to the result, I convolved each constraint separately with Eq. 2 (Fig. 5). The result does not depend strongly on any one of the constraints (see "all - x" results in Table 2). For example, the age, independent of the SNe data, is $t_o(\text{all} - \text{SNe}) = 13.3^{+1.7}_{-1.8}$ Ga, which differs negligibly from the main result. The

age, independent of the SNe and CMB data, is $t_o(\text{all} - \text{CMB} - \text{SNe}) = 12.6^{+3.4}_{-2.0}$ Ga, which is somewhat lower than the main result but within the error bars.

The Oldest Objects in Our Galaxy

The universe cannot be younger than the oldest objects in it. Thus, estimates of the age of the oldest objects in our Galaxy are lower limits to the age of the universe (Table 2 and Fig. 2). A standard but simplified scenario for the origin of our Galaxy has a halo of globular clusters forming first, followed by the formation of the Galactic disk. The most recent measurements of the age of the oldest objects in the Galactic disk give $t_{\text{disk}} = 8.7 \pm 0.4$ Ga (Table 2). The most recent measurements of the age of the oldest objects in the halo of our Galaxy give $t_{\text{Gal}} = 12.2 \pm 0.5$ Ga (Table 2). The individual measurements are in good agreement with these averages. There are no large outliers. In contrast to the $t_o(h, \Omega_m, \Omega_\Lambda)$ estimates obtained above, all of these age estimates are direct in the sense that they have no dependence on a Big Bang model.

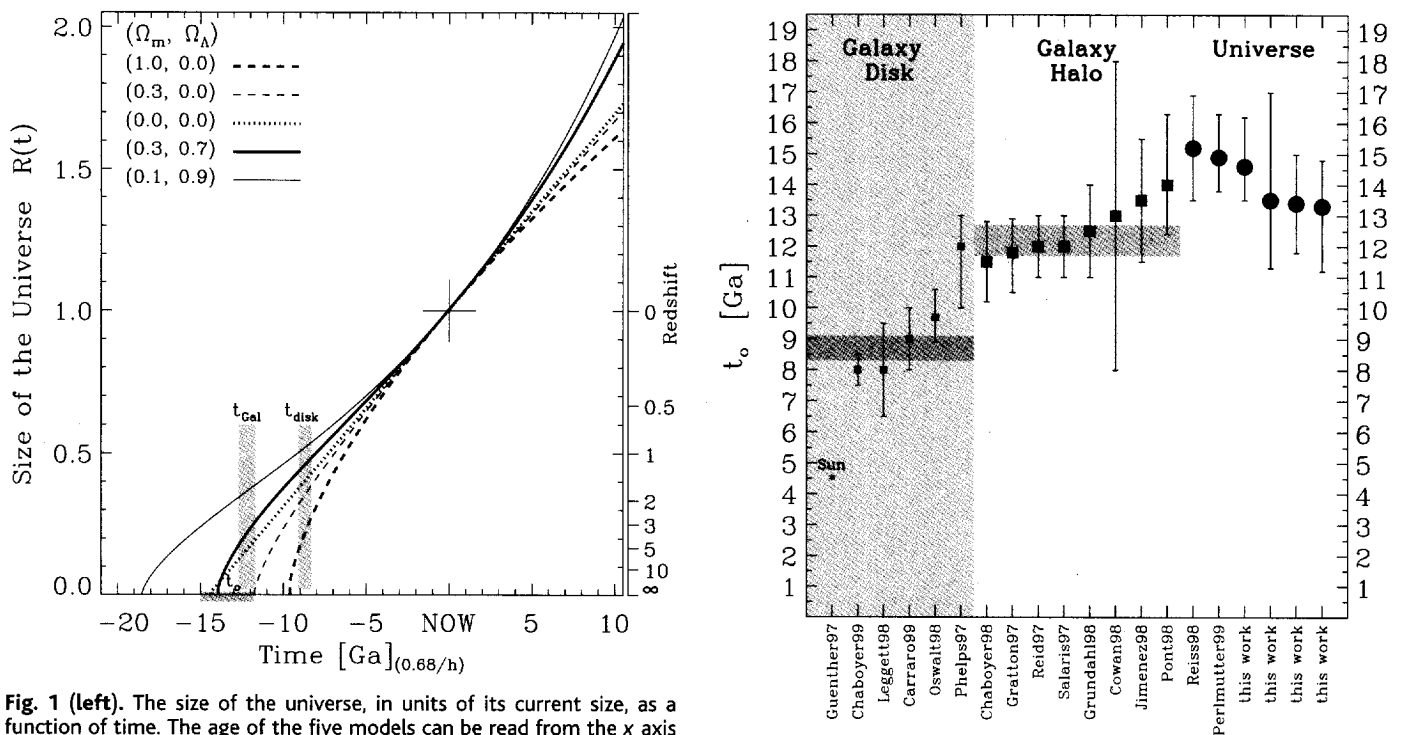


Fig. 1 (left). The size of the universe, in units of its current size, as a function of time. The age of the five models can be read from the x axis as the time between NOW and the intersection of the model with the x axis. The main result of this paper, $t_o = 13.4 \pm 1.6$ Ga, is labeled "t_o" and is shaded gray on the x axis. Measurements of the age of the halo of our Galaxy yield $t_{\text{Gal}} = 12.2 \pm 0.5$ Ga, whereas measurements of the age of the disk of our Galaxy yield $t_{\text{disk}} = 8.7 \pm 0.4$ Ga (Table 2). These age ranges are also labeled and shaded gray. The $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$ model fits the constraints of Table 1 better than the other models shown. Over the past few billion years and on into the future, the rate of expansion of this model increases ($\ddot{R} > 0$). This acceleration means that we are in a period of slow inflation. Other consequences of a Ω_Λ -dominated universe are discussed in (50). On the x axis, $h = 0.68$ has been assumed. For other values of h , multiply the x axis ages by $0.68/h$. Redshifts are indicated on the right. **Fig. 2 (right).** Age estimates of the universe and of the oldest objects in our Galaxy. The four estimates of the age of the universe from this work are indicated in Table 2. The

three similar points near 13.4 Ga result from $h = 0.64, 0.68,$ and 0.72 and indicate that the result is not strongly dependent on h when a reasonable h uncertainty of ± 0.10 is used. Among the four, the highest value at 14.6 Ga comes from assuming $h = 0.64 \pm 0.02$. All the estimates in the top section of Table 2 are plotted here. As in Fig. 1, averages of the ages of the Galactic halo and Galactic disk are shaded gray. The absence of any single age estimate more than $\sim 2\sigma$ from the average adds plausibility to the possibly overdemocratic procedure of computing the variance-weighted averages. The result that $t_o > t_{\text{Gal}}$ is logically inevitable, but the standard Einstein-deSitter model does not satisfy this requirement unless $h < 0.55$. The reference for each measurement is given under the x axis. The age of the sun is accurately known and is included for reference. Error bars indicate the reported 1σ limits.

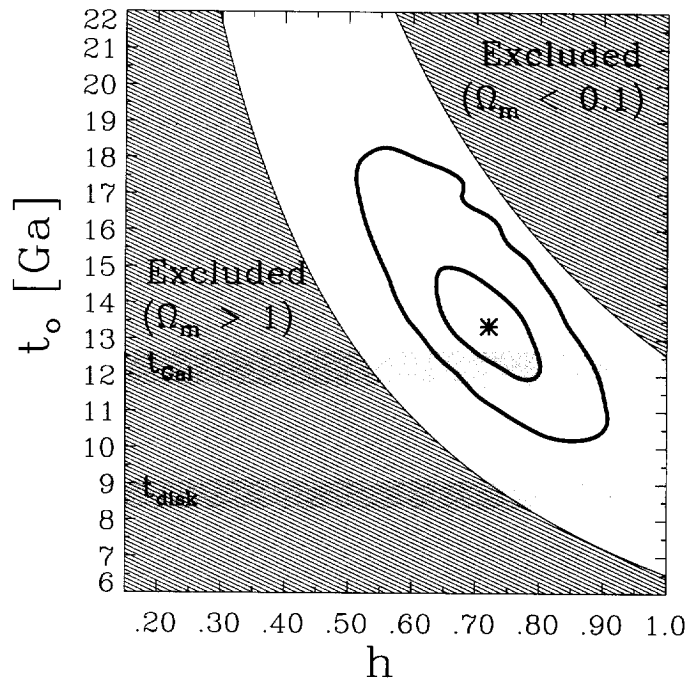
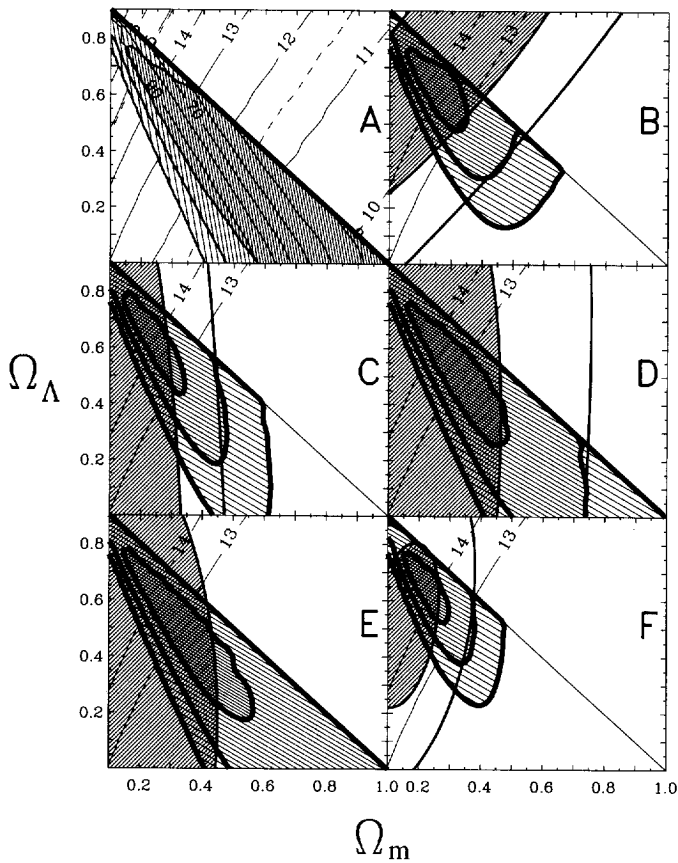


Fig. 3 (left). The regions of the $(\Omega_m, \Omega_\Lambda)$ plane preferred by various constraints. (A) Cosmic microwave background, (B) SNe, (C) cluster mass-to-light ratios, (D) cluster abundance evolution, (E) double radio lobes, and (F) all combined. The power of combining CMB constraints with each of the other constraints (Table 1) is also shown. The elongated areas (from upper left to lower right) in (A) are the approximate 1σ , 2σ , and 3σ confidence levels of the likelihood from CMB data, L_{CMB} (9). (A) also shows the important h dependence of L_{CMB} . The contours within the dark shaded region are of h values that maximize L_{CMB} for a given $(\Omega_m, \Omega_\Lambda)$ pair ($h = 0.70$ and 0.90). This correlation between preferred h and preferred $(\Omega_m, \Omega_\Lambda)$ helps $L_{\text{CMB}}(h, \Omega_m, \Omega_\Lambda)$ constrain t_0 . In (B) through (E), thin contours enclose the 1σ (shaded) and 2σ confidence regions from separate constraints, and thick contours indicate the 1σ , 2σ , and 3σ regions of the combination of L_{CMB} with these same constraints. (F) shows the region preferred by the combination of the separate constraints shown in (B) through (E) (thin contours) as well as the combination of (A) through (E) (thick contours). The best fit values are $\Omega_\Lambda = 0.65 \pm 0.13$ and $\Omega_m = 0.23 \pm 0.08$. In (A), the thin iso- t_0 contours (labeled "10" through "14") indicate the age (billion years ago) when $h = 0.68$ is assumed. For reference, the 13- and 14-Ga contours are in all panels. To give an idea of the sensitivity of the h dependence of these contours, the two additional dashed contours in (A) show the 13-Ga contours for $h = 0.58$ and 0.78 (the 1σ limits of the principle h estimate used in this paper). In (F), it appears that the best fit has $t_0 \approx 14.5$ Ga, but all constraints shown here are independent of information about h ; they do not include the h dependence of L_{CMB} , L_{baryons} , or L_{Hubble} (Table 1).

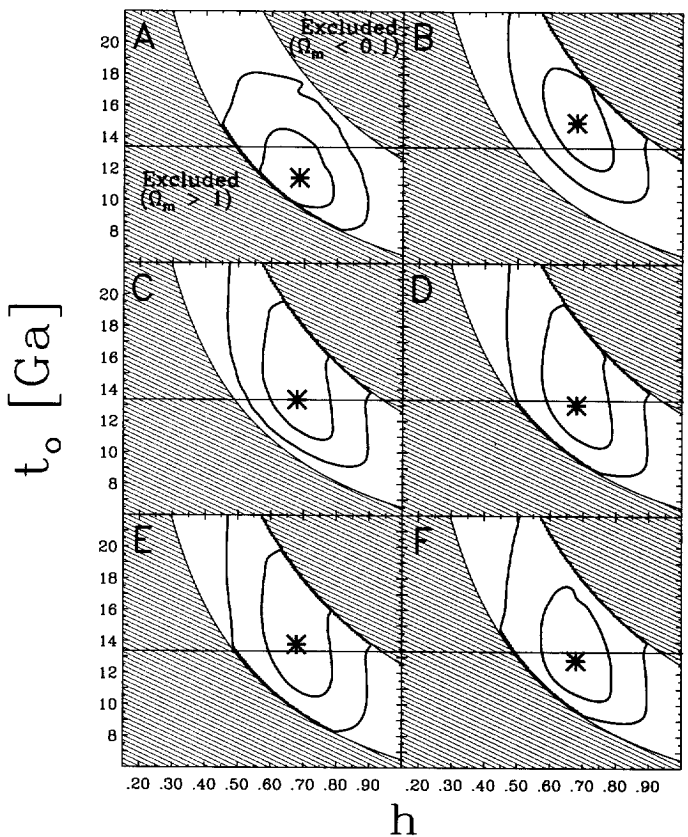


Fig. 4 (top right). This plot shows the region of the $h - t_0$ plane preferred by the combination of all seven constraints. The result, $t_0 = 13.4 \pm 1.6$ Ga, is the main result of this paper. The thick contours around the best fit (indicated by a star) are at likelihood levels defined by $L/L_{\text{max}} = 0.607$ and 0.135 , which approximate 68 and 95% confidence levels, respectively. These contours can be projected onto the t_0 axis to yield the age result. This age result is robust to variations in the Hubble constraint as indicated in Table 2. The areas marked "Excluded" (here and in Fig. 5) result from the range of parameters considered: $0.1 \leq \Omega_m \leq 1.0$ and $0 \leq \Omega_\Lambda \leq 0.9$ with $\Omega_m + \Omega_\Lambda \leq 1$. Thus, the upper (high t_0) boundary is defined by $(\Omega_m, \Omega_\Lambda) = (0.1, 0.9)$, and the lower boundary is the standard Einstein-deSitter model defined by $(\Omega_m, \Omega_\Lambda) = (1, 0)$. Both of these boundary models are plotted in Fig. 1. The estimates from Table 2 of the age of our Galactic halo (t_{Gal}) and the age of the Milky Way (t_{disk}) are shaded grey. The universe is about 1 billion years older than our Galactic halo. The combined constraints also yield a best fit value of the Hubble constant which can be read off of the x axis ($h = 0.73 \pm 0.09$, a slightly higher and tighter estimate than the input $h = 0.68 \pm 0.10$).

Fig. 5 (bottom right). The purpose of this figure is to show how Fig. 4 is built up from the seven independent constraints used in the analysis. All six panels are analogous to Fig. 4 but contain only the Hubble constraint [$h = 0.68 \pm 0.10$ (Eq. 2)] convolved with a single constraint: (A) cosmic microwave background, (B) SNe, (C) cluster mass-to-light ratios, (D) cluster abundance evolution, (E) double radio lobes, and (F) baryons (Table 1). The relative position of the best fit (indicated by a star) and the 13.4-Ga line indicates how each constraint contributes to the result.

How old was the universe when our Galaxy formed? If we write this as $t_{\text{Gal}} + \Delta t = t_o$, then what is the amount of time (Δt) between the formation of our Galaxy and the formation of the universe? If we had an estimate of Δt , then we would have an independent estimate of t_o to compare to $t_o = 13.4 \pm 1.6$ Ga, obtained above. However, we have very poor constraints on Δt . The simple but plausible estimate $\Delta t \approx 1$ Gy is often invoked, but estimates range from ~ 0.1 to ~ 5 Gy and may be even larger (24, 25). This uncertainty in Δt undermines the ability of estimates of the age of the oldest objects in our Galaxy to tell us the age of the universe. Without Δt , we cannot infer t_o from t_{Gal} . The best estimate of Δt may come from the difference

between the age reported here and the estimate of the age of our Galaxy (Table 2). Thus, $\Delta t = t_o - t_{\text{Gal}} = 13.4 - 12.2 = 1.2 \pm 1.8$ Gy.

The age measurements in Table 2 also indicate that there is a 3.5-Gy period between halo and disk formation ($t_{\text{Gal}} - t_{\text{disk}}$). If our Milky Way is typical, then this may be true of other spiral galaxies. With the best fit values obtained here for the three parameters, ($h, \Omega_m, \Omega_\Lambda$) = (0.72 \pm 0.09, 0.23 \pm 0.08, 0.65 \pm 0.13), the ages t_{disk} and t_{Gal} can be converted into the redshifts at which the disk and halo formed: $z_{\text{disk}} = 1.3^{+1.5}_{-0.5}$ and $z_{\text{Gal}} = 6.0^{+0.3}_{-4.3}$. Thus, a diskless epoch should be centered at a redshift between z_{disk} and z_{Gal} ($1.3 \approx z_{\text{diskless}} \approx 6.0$). We would expect

fewer disks in the halolike progenitors of spiral galaxies in this redshift range. Studies of galaxy types in the Hubble Deep Field indicate that this may be the case (26).

The requirement that the universe be older than our Galaxy ($t_o > t_{\text{Gal}}$) is a consistency test of the Big Bang model. The best fit model obtained here passes this test. There is no age crisis. This is true even if the high values of h (~ 0.80) are correct. Only at $h \approx 0.85$ is $t_o \approx t_{\text{Gal}}$. This consistency provides further support for the Big Bang model, which the standard model ($\Omega_m = 1, \Omega_\Lambda = 0$) is unable to match unless $h < 0.55$.

Comparison with Previous Work

The goal of this paper is to determine the absolute age of the universe $t_o(h, \Omega_m, \Omega_\Lambda)$. Knowledge of h alone cannot be used to determine t_o with much accuracy. For example, the estimate $h = 0.68 \pm 0.10$ corresponds to $8 < t_o < 22$ Ga (Fig. 4). Similarly, knowledge of (Ω_m, Ω_Λ) yields $H_o t_o(\Omega_m, \Omega_\Lambda)$, not t_o (H_o is the usual Hubble constant). When one inserts a preferred value of h into a $H_o t_o$ result, one is not taking into consideration the correlations between preferred h values and preferred (Ω_m, Ω_Λ) values that are inherent, for example, in $L_{\text{CMB}}(h, \Omega_m, \Omega_\Lambda)$ and $L_{\text{baryons}}(h, \Omega_m)$. The preferred values of h in these likelihoods depend on Ω_m and Ω_Λ . Perlmutter *et al.* (4) used SNe measurements to constrain (Ω_m, Ω_Λ) and obtained values for $H_o t_o$. To obtain t_o , they did the analysis with h set equal to the value preferred by their SNe data, $h = 0.63$. Their result is $t_o = 14.5 \pm 1.0(0.63/h)$ Ga. When a flat universe is assumed, they obtain $t_o^{\text{flat}} = 14.9^{+1.4}_{-1.1}(0.63/h)$ Ga. Riess *et al.* (5) found $h = 0.65 \pm 0.02$ from their SNe data. Marginalizing over this Hubble value and over Ω_Λ and Ω_m , they report $t_o = 14.2 \pm 1.7$ Ga. When a flat universe is assumed, their results yield $t_o^{\text{flat}} = 15.2 \pm 1.7$ Ga. The Perlmutter *et al.* (4) and Riess *et al.* (5) results are in good agreement. When I assume $h = 0.64 \pm 0.02$, I get $t_o = 14.6^{+1.6}_{-1.1}$ Ga. This result is plotted in Fig. 2 to illustrate the important influence on the result of using a small h uncertainty. Efstathiou *et al.* (12), on the basis of a combination of CMB and Perlmutter *et al.* (4) SNe data, have estimated $t_o = 14.6(h/0.65)^{-1}$ Ga. I used $h = 0.65 \pm 0.0$ with this data combination to get $t_o = 14.5^{+1.2}_{-1.0}$ Ga. However, when I used $h = 0.65 \pm 0.10$, the result is 0.7 Gy lower ($t_o = 13.8^{+3.2}_{-1.4}$ Ga). To obtain the main result, I used uncertainties large enough to reflect our knowledge of h , on the basis of many sources. The use of a larger h uncertainty contributes to the substantially younger ages found here (23).

A potential problem with the SNe ages is the high region, (Ω_m, Ω_Λ) \approx (0.8, 1.5), which dominates the SNe fit. This region is strongly disfavored by the six other constraints considered here (see Fig. 3). These high (Ω_m, Ω_Λ) values allow lower ages than the t_o^{flat} SNe re-

Table 2. Age estimates of our Galaxy and universe (36). "Technique" refers to the method used to make the age estimate. OC, open clusters; WD, white dwarfs; LF, luminosity function; avg, average; GC, globular clusters; M/L, mass-to-light ratio; and cl evol, cluster abundance evolution. The averages are inverse variance-weighted averages of the individual measurements. The sun is not included in the disk average. "Isotopes" refers to the use of relative isotopic abundances of long-lived species as indicated by absorption lines in spectra of old disk stars. The "stellar ages" technique uses main sequence fitting and the new Hipparcos subdwarf calibration. "All" means that all six constraints in Table 1 and the CMB constraints were used in Eq. 1. "All - x" means that all seven constraints except constraint x were used in Eq. 1. Figures 3 and 5 and the all - x results indicate a high level of agreement between constraints and the lack of dependence on any single constraint. Thus, there is a broad consistency between the ages preferred by the CMB and the six other independent constraints. Figure 2 presents all of the disk and halo age estimates.

Technique	Reference	h assumptions	Age (Ga)	Object
Isotopes	(37)	None	4.53 \pm 0.04	Sun
Stellar ages	(38)	None	8.0 \pm 0.5	Disk OC
WD LF	(39)	None	8.0 \pm 1.5	Disk WD
Stellar ages	(40)	None	9.0 \pm 1	Disk OC
WD LF	(25)	None	9.7 ^{+0.9} _{-0.8}	Disk WD
Stellar ages	(41)	None	12.0 ^{+1.0} _{-2.0}	Disk OC
		None	8.7 \pm 0.4	t_{disk} (avg)
Stellar ages	(42)	None	11.5 \pm 1.3	Halo GC
Stellar ages	(43)	None	11.8 ^{+1.1} _{-1.3}	Halo GC
Stellar ages	(44)	None	12 \pm 1	Halo GC
Stellar ages	(45)	None	12 \pm 1	Halo GC
Stellar ages	(46)	None	12.5 \pm 1.5	Halo GC
Isotopes	(47)	None	13.0 \pm 5	Halo stars
Stellar ages	(48)	None	13.5 \pm 2	Halo GC
Stellar ages	(49)	None	14.0 ^{+2.3} _{-1.6}	Halo GC
		None	12.2 \pm 0.5	t_{Gal} (avg)
SNe	(4)	0.63 \pm 0.0	14.5 \pm 1.0	Universe
SNe (flat)	(4)	0.63 \pm 0.0	14.9 ^{+1.4*} _{-1.1}	Universe
SNe	(5)	0.65 \pm 0.02	14.2 \pm 1.7	Universe
SNe (flat)	(5)	0.65 \pm 0.02	15.2 \pm 1.7*	Universe
All	This work	0.60 \pm 0.10	15.5 ^{+2.3} _{-2.8}	Universe
All	This work	0.64 \pm 0.10	13.5 ^{+3.5*} _{-2.2}	Universe
All	This work	0.68 \pm 0.10	13.4 ^{+1.6*} _{-1.6}	Universe
All	This work	0.72 \pm 0.10	13.3 ^{+1.2*} _{-1.4}	Universe
All	This work	0.76 \pm 0.10	12.3 ^{+1.9} _{-1.6}	Universe
All	This work	0.80 \pm 0.10	11.9 ^{+1.9} _{-1.6}	Universe
All	This work	0.64 \pm 0.02	14.6 ^{+1.6*} _{-1.1}	Universe
All - CMB	This work	0.68 \pm 0.10	14.0 ^{+3.0} _{-2.2}	Universe
All - SNe	This work	0.68 \pm 0.10	13.3 ^{+1.9} _{-1.8}	Universe
All - M/L	This work	0.68 \pm 0.10	13.3 ^{+1.9} _{-1.7}	Universe
All - cl evol	This work	0.68 \pm 0.10	13.3 ^{+1.7} _{-1.4}	Universe
All - radio	This work	0.68 \pm 0.10	13.3 ^{+1.7} _{-1.5}	Universe
All - baryons	This work	0.68 \pm 0.10	13.4 ^{+2.6} _{-1.5}	Universe
All - Hubble	This work	None	< 14.2	Universe
All - CMB - SNe	This work	0.68 \pm 0.10	12.6 ^{+3.4} _{-2.0}	Universe

*Also plotted in Fig. 2.

sults because the slope of the iso- t_0 contours (Fig. 3B) is larger than the slope of the SNe contours. The t_0^{flat} results are not as subject to this problem and are the results most analogous to the result reported here, despite the fact that the SNe t_0^{flat} results are less consistent with the result reported here. There are several independent cosmological measurements that have not been included in this analysis either because a consensus has not yet been reached [gravitational lensing limits (27–30)] or because the analysis of the measurements has not been done in a way that is sufficiently free of conditioning on certain parameters [local velocity field limits (31)]. Doubts about some of the observations used here are discussed in (32). There has been speculation recently that the evidence for Ω_Λ is really evidence for some form of stranger dark energy that we have been incorrectly interpreting as Ω_Λ . Several workers have tested this idea. The evidence so far indicates that the cosmological constant interpretation fits the data as well as or better than an explanation based on more mysterious dark energy (4, 33, 34).

References and Notes

- I follow a common convention and work with dimensionless quantities. The dimensionless Hubble constant is $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, where H_0 is the usual Hubble constant in units of kilometers per second per megaparsec. Both Ω_m and Ω_Λ are densities expressed in units of the critical density, which is defined as $\rho_{\text{crit}} = 3H_0^2/8\pi G$, where G is Newton's constant. Thus, $\Omega_m = \rho_m/\rho_{\text{crit}}$ where ρ_m is the mass density of the universe in grams per cubic centimeter. The dimensionless cosmological constant $\Omega_\Lambda = \rho_\Lambda/\rho_{\text{crit}} = \Lambda/3H_0^2$, where Λ is the cosmological constant introduced in 1917 [A. Einstein, *Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl.* **1917**, 142 (1917); English translation in *Principle of Relativity*, H. A. Lorentz et al., Eds. (Dover, New York, 1952), pp. 175].
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- Galaxy cluster mass-to-light ratio limits are from R. Carlberg et al. [*Astrophys. J.* **478**, 462 (1997)] and R. Carlberg et al. [paper presented at the 33rd Recontres de Moriond, Fundamental Parameters in Cosmology, Les Arcs, France, 17 to 24 January, 1998 (available at <http://xxx.lanl.gov/abs/astro-ph/9804312>)]. The error bars in Table 1 are conservative in the sense that they include their "worst case," 73% errors.
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- Baryonic fraction of the mass of clusters of galaxies. Evrard (14) reports $\Omega_m/\Omega_b = (12.5 \pm 0.7)h^{4/3}$ (independent of Ω_Λ), which becomes $\Omega_m/\Omega_b = (12.5 \pm 3.8)h^{4/3}$ when the suggested 30% systematic error is added in quadrature. Burles and Tytler (15) report $\Omega_b h^2 = 0.019 \pm 0.001$, but Fukugita et al. (18) favor a lower but larger range that includes $\Omega_b h^2 = 0.020$. I adopt $\Omega_b h^2 = 0.015 \pm 0.005$. When this is combined with the Evrard (14) result, one gets the constraint $\Omega_m h^{2/3} = 0.19 \pm 0.12$.
- The remaining five L_i have no simple analytic form.
- These combinations prefer the upper left region of the $(\Omega_m, \Omega_\Lambda)$ plane. I (9) have analyzed this complementarity and obtained $\Omega_\Lambda = 0.62 \pm 0.16$ and $\Omega_m = 0.24 \pm 0.10$. The addition of the cluster abundance evolution measurements and improvements in the SNe measurements tightens these limits to the values given.
- $L(h, \Omega_m, \Omega_\Lambda)$ was derived as in (9). For a discussion of taking the maximum of the likelihood, rather than performing an integral to marginalize over the nuisance variable, see (3, p. 626) and (11, Section 2.4) and references therein. The lower right of Fig. 3A shows that low h values are preferred (which correspond to older ages because $t_0 \propto 1/h$). This preference contrasts with the iso- t_0 lines (Fig. 3A), which have the younger ages in the lower right. Therefore, the three-dimensional CMB preferences will yield tighter limits on t_0 than is apparent from the figure.
- In the absence of any direct constraint on h , the data are more consistent with large values of h . For example, if no $L_{\text{Hubble}}(h)$ is used in Eq. 1, the combined data yield a lower limit $h > 0.67$. The corresponding age limit is $t_0 < 14.2$ Ga. No upper limit could be determined because the best fit for this data is $h = 1.00$, at the edge of the range explored. This preference for higher h values (lower ages) helps explain the h dependence of the results in Table 2. Results for $0.60 \leq h \leq 0.80$ are reported in Table 2. When a small uncertainty in h was assumed (effectively conditioning on an h value), it dominated the h information from the other constraints. For example, although the central h values are the same, the results from using $h = 0.64 \pm 0.02$ and $h = 0.64 \pm 0.10$ are quite different: 14.6
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- I use SNe constraints large enough to encompass the constraints from the two SNe groups. Perlmutter et al. (4) report $0.8(\Omega_m) - 0.6(\Omega_\Lambda) = -0.2 \pm 0.1$. Evaluated at $\Omega_\Lambda = 0$, this yields $\Omega_m^{\Lambda=0} = -0.25 \pm 0.13$. Assuming spatial flatness, they report $\Omega_m^{\text{flat}} = 0.28^{+0.09(+0.05)}_{-0.08(-0.04)}$ statistical (systematic) errors. Adding the statistical and systematic errors in quadrature yields $\Omega_m^{\text{flat}} = 0.28^{+0.10}_{-0.09}$. The Riess et al. (5) constraints are from their figures 6 and 7 ("MLCS method" and "snapshot method"), using either the solid or dotted contours, whichever is larger (corresponding to the analysis with and without SN1997ck, respectively). This yields $\Omega_m^{\Lambda=0} = -0.35 \pm 0.18$ and $\Omega_m^{\text{flat}} = 0.24^{+0.17}_{-0.10}$. I use the weighted average of these Perlmutter et al. (4) and Riess et al. (5) constraints to obtain the SNe constraints listed above. The crucial upper error bars on Ω_m are large enough to include the constraints from either reference.
- One would like to be less Galactocentric, but measuring the ages of even nearby extragalactic objects is difficult. See K. A. Olsen et al. [*Mon. Not. R. Astron. Soc.* **300**, 665 (1998)], who find that the ages of old globular clusters in the Large Magellanic Cloud are the same age as the oldest Galactic globular clusters.
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