

DARK-ENERGY DYNAMICS REQUIRED TO SOLVE THE COSMIC COINCIDENCE

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ABSTRACT

Many exotic dark energy models are designed to solve the cosmic coincidence (why, just now is $\Omega_{de} \sim \Omega_m$?) by guaranteeing $\Omega_{de} \sim \Omega_m$ for significant fractions of the age of the universe. This typically entails purpose-built tracking or oscillatory behaviour in the model. However, such measures are neither required for, nor guarantee, a satisfactory solution. Instead what must be shown is that $\Omega_{de} \sim \Omega_m$ for a significant fraction of observers in the universe. We explore the consequences of this by making a simple estimate of the temporal distribution of observers. Our main result is simple: any model fitting current observational constraints on ρ_{de} , w_0 and w_a does have $\Omega_{de} \sim \Omega_m$ for a large fraction of observers in the universe. This may demotivate DDE models specifically designed to generate long or repeated periods of $\Omega_{de} \sim \Omega_m$, and should help to simplify the considerations that go into future DDE models. The dynamical requirements explored here are necessary but not sufficient to solve the cosmic coincidence using DDE - theorists must still seek to avoid fine-tuning in their models.

Subject headings:

1. INTRODUCTION

Even before 1998 there were good reasons for thinking that our cosmological picture was only 5% complete. Inflation generically predicts that the universe is spatially flat, $\rho_{tot} \approx \rho_{crit} = 3H^2/8\pi G$, yet estimates of the dynamic matter density were set at just 30% of that required for flatness, $\Omega_m = \rho_m/\rho_{crit} \approx 0.3$ (Ostriker and Steinhardt 1995). Moreover, big bang nucleosynthesis constraints meant that just a sixth of the dynamic matter density could be accounted for by the baryons in stars and gas, $\Omega_b = \rho_b/\rho_{crit} \approx 0.05$ (Walker et al. 1991).

In 1998, using supernovae Ia as calibrated candles, Riess et al. (1998) and Perlmutter et al. (1999) made direct observations of the expansion history of the universe and revealed a recent and continuing epoch of cosmic acceleration. Here was conclusive evidence that Einstein's cosmological constant Λ , or something else with comparable negative pressure, currently dominates the energy density of the universe (Lineweaver 1998). Λ identifies physically with the energy of zero-point quantum fluctuations in the vacuum (Sakharov REF) and, with a constant equation of state $w \equiv p_{de}/\rho_{de} = -1$, is the simplest way to explain the observations. This necessary additional energy component, interpreted as Λ or otherwise, has become generally known as the "dark energy".

A plethora of observations since then have been used to constrain the free parameters of the orthodox cosmological model, Λ CDM, in which Λ *does* play the role of the dark energy. Hinshaw (2006) find that the universe is expanding at a rate of $H_0 = 71 \pm 4$ Km/s/Mpc; that it is flat therefore it is critically dense ($\Omega_{tot0} = [8\pi G/3H_0^2]\rho_{tot0} = 1.01 \pm 0.01$); and that the total density is comprised of contributions from radiation ($\Omega_{r0} = 4.5 \pm 0.2 \times 10^{-5}$), baryonic matter ($\Omega_{b0} = 0.044 \pm 0.003$),

cold dark matter (CDM; $\Omega_{CDM0} = 0.22 \pm 0.02$), and vacuum energy ($\Omega_{\Lambda0} = 0.74 \pm 0.02$). Henceforth we will assume that the universe is exactly flat $\Omega_{tot0} = 1$ as predicted by inflation.

Two notorious problems have been very influential in moulding our ideas about dark energy, specifically in driving interest in alternatives to Λ CDM. The first of these problems is concerned with the smallness of the dark energy density. Despite representing more than 70% of the total energy of the universe, the current dark energy density is 120 orders of magnitude smaller than an equipartition of the available energy at the end of inflation (80 orders of magnitude smaller if this occurred at the GUT rather than Planck scale). Dark energy candidates are thus challenged to explain how the current small value eventuates.

The second cosmological constant problem is concerned with the near coincidence between the current cosmological matter density ($\rho_{m0} \approx 0.26 \times \rho_{crit0}$) and the dark energy density ($\rho_{de0} \approx 0.74 \times \rho_{crit0}$). In the standard Λ CDM model, the cosmological window during which these components have comparable density is short (just 1.5 e-folds of the cosmological scalefactor a) since the matter density dilutes as $\rho_m \propto a^{-3}$ while the vacuum density ρ_{de} is constant. Thus even if one explains why the DE density is much less than planck energies (the smallness problem) one must explain how we happen to live during the time when $\rho_{de} \sim \rho_m$. To quantify the time-dependent proximity of ρ_m and ρ_{de} we define "the proximity parameter",

$$r \equiv \min \left[\frac{\rho_{de}}{\rho_m}, \frac{\rho_m}{\rho_{de}} \right], \quad (1)$$

which ranges from $r \approx 0$, when many orders of magnitude separate the two densities, to $r = 1$, when the two densities are equal. The presently observed value of this parameter is $r_0 = \frac{\rho_{m0}}{\rho_{de0}} \approx 0.35$. The coincidence problem is illustrated in figure 1. In terms of r , the coincidence problem is as follows. If we let the time of observation t_0 vary across many orders of magnitude we find that the

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expectation proximity is $r \approx 0 \ll 0.35$.

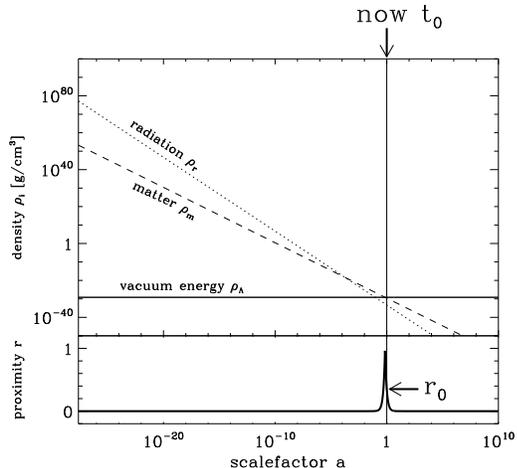


FIG. 1.— This figure shows the history of the energy density of the universe according to concordance Λ CDM. The dotted line shows the energy density in radiation (photons, neutrinos and other relativistic modes). The radiation density dilutes as a^{-4} as the universe expands. The dashed line shows the density in ordinary non-relativistic matter, which dilutes as a^{-3} . The solid line shows the energy of the vacuum (the cosmological constant) which has remained constant since the end of inflation. The lower panel shows the proximity r of the matter density to the vacuum energy density (see Eq. 1). The proximity r is only large for a brief moment in the $\log(a)$ history of the cosmos. The coincidence problem is that matter and density happen to have very similar densities (r is large) at the current time t_0 .

Weinberg (1987); Garriga et al. (1999); Lineweaver and Egan (2007) and others have argued that the coincidence problem can be made quantitatively meaningful if the hypothetical variability of t_0 is limited to values allowed by the temporal distribution of terrestrial planets. In (Lineweaver and Egan 2007) we assessed the severity of the coincidence problem under Λ CDM and demonstrate that the observed proximity $r \approx 0.35$ is likely for terrestrial-planet bound observers. It may be the case that future developments in fundamental physics reveal the cosmological constant to be uniquely determined. Lineweaver and Egan (2007) shows that the smallness and coincidence problems would then be simultaneously solved.

The smallness of the DE density may be understood in the context of multiverse models in which ρ_{de} is a stochastic variable. The smallness of the observed value is explained because much larger values preclude the formation of galaxies - those universes are devoid of observers and are anthropically selected against (Weinberg 1987; Martel et al. 1998; Pogosian and Vilenkin 2007). The solution to the coincidence problem in these scenarios was given by (Garriga et al. 1999). Although these solutions are attractive, the theoretical foundations for such multiverses are not well understood, and these solutions may be altogether inappropriate.

Dynamical dark energy (DDE) models including quintessence, phantom, k-essence, Chaplygin gas and others have also had some success simultaneously tackling the smallness and coincidence problems. In these models the dark energy is treated as a new matter field which is effectively homogenous, and evolves as the universe expands. Some examples from the literature are given in section 2. Many DDE models are designed to

ensure that $\rho_{de} \sim \rho_m$ for a large fraction of the history/future of the universe (Dodelson et al. 2000; Sahni and Wang 2000; Chimento et al. 2000; Zimdahl et al. 2001; Sahni 2002; Chimento et al. 2003; Ahmed et al. 2004; França and Rosenfeld 2004; Mbonye 2004; Guo and Zhang 2005; Pavón and Zimdahl 2005; Scherrer 2005; Zhang 2005; França 2006; Feng et al. 2006; Nojiri and Odintsov 2006; Amendola et al. 2006, 2007; Sassi and Bonometto 2007). This is at least partly motivated to avoid the coincidence³ - by having $\rho_{de} \sim \rho_m$ during extended or repeated periods one may hope to ensure that $r = O(1)$ is the expectation. Precisely when, and for how long, must a DDE model have $\rho_{de} \sim \rho_m$ in order to solve the coincidence? A carefully considered discussion of this has not yet been given.

Thus the general goal of the present paper is to determine which dynamical behaviours are required to solve the coincidence problem, and which are unnecessary, based on an estimate of the temporal distribution of terrestrial planets. In particular we ask the question, “Does a dark energy model naturally fitting contemporary constraints on the density ρ_{de} and the equation of state parameters, necessarily solve the cosmic coincidence?” Both positive and negative answers have interesting consequences. An answer in the affirmative will simplify considerations that go into DDE modeling; the coincidence is solved by all models naturally fitting cosmological constraints. An answer in the negative would announce a unique and peculiar opportunity to constrain the DE equation of state parameters *more strongly* than contemporary cosmological surveys.

In Section 2 we present several examples of how dark energy dynamics have been used to solve the coincidence problem. Current observational constraints on dark energy dynamics are discussed in Section 3. We discuss the treatment of anthropic observational selection effects in Section 4 and estimate the relevant temporal distribution of observers in Section 5. Our main analysis and the results are presented in Section 6 and we conclude in Section 7.

2. DYNAMIC DARK ENERGY MODELS IN THE FACE OF THE COSMIC COINCIDENCE

There are many interesting DDE models in the literature. Though it is beyond the scope of this article to provide a complete review, this section looks at a few representative examples in order to set the context and motivation of our work. Figure 2 illustrates density histories typical of quintessence, tracking oscillating energy, k-essence, interacting quintessence, phantom fields, late-time scaling DE, and Chaplygin gas. They are discussed in turn below, then the main points are summarized.

The *most* relevant references are given. Readers seeking more detail are referred to the excellent review article

³ It is worth noting that in the context of DDE models “the coincidence problem” sometimes refers to difficulties where the model must be fine-tuned or constructed in an ad-hoc manner in order to fit cosmological observations of the present densities ρ_{de} , ρ_m and DE equation of state w_{de} (let us refer to these as fine-tuning problems henceforth). We don’t address fine-tuning problems (you may want to see (Bludman 2004; Linder 2006)). What we refer to as the coincidence problem is when, according to the model, an observer has to be special amongst hypothetical observers populating the model, in order to observe r as large as we do (i.e. $r \gtrsim 0.35$).

by Copeland et al. (2006) and references therein.

2.1. Tracker Quintessence

In quintessence models the dark energy is interpreted as a homogenous minimally-interacting scalar field permeating the universe (Özer and Taha 1987; Ratra and Peebles 1988; Ferreira and Joyce 1998; Caldwell et al. 1998; Steinhardt et al. 1999; Zlatev et al. 1999; Dalal et al. 2001). The evolution of the quintessence field and of the cosmos depends on the postulated potential $V(\phi)$ of the field. In general, quintessence has a time-varying equation of state restricted to values $w > -1$. Particular choices for $V(\phi)$ lead to interesting attractor solutions which can be exploited to make ρ_{de} scale sub-dominantly until recently when the quintessence field transits to a Λ -like state. In this state it dominates the energy of the universe and drives de-Sitter-like acceleration (in agreement with observations). We illustrate these behaviours in Figure 2 using two simple quintessence potentials $V(\phi) = M\phi^{-\alpha}$ (light blue) and $V(\phi) = M \exp(1/\phi)$ (dark blue). We start both these models in tracker states⁴ and tune M in each model so as to match the DE density ρ_{de0} observed today.

2.2. Oscillating Dark Energy

Dodelson et al. (2000) explored a quintessence potential with oscillatory perturbations $V(\phi) = M \exp(-\lambda\phi) [1 + A \sin(\nu\phi)]$. They refer to models of this type as tracking oscillating energy. Without the perturbations (setting $A = 0$) this potential causes exact tracker behaviour: the quintessence energy decays as ρ_{r+m} and never dominates. With the perturbations the quintessence energy density oscillates about ρ_{r+m} as they decay. See the example (the red line) in Figure 2. The quintessence energy dominates on multiple occasions and its equation of state varies continuously between positive and negative values. One of the main motivations for tracking oscillating energy is to solve the coincidence problem by ensuring that any past or future observer would also see $\rho_{de} \sim \rho_m$ or $\rho_{de} \sim \rho_r$. The parameters of the model (for this particular $V(\phi)$) they are M , λ , A and ν require minimal fine-tuning. On the other hand, it has yet to be seen how such a potential might arise from particle physics. Phenomenologically similar cosmologies have been discussed in (Ahmed et al. 2004; Yang and Wang 2005; Feng et al. 2006).

Chimento et al. (2000, 2003); França and Rosenfeld (2004); Pavón and Zimdahl (2005); França (2006); Amendola et al. (2006, 2007); Sassi and Bonometto (2007) and others have explored models in which the DDE oscillates about the matter density and asymptotically approaches a scaling solution with $r_\infty \approx 0.35$. This late-time scaling behaviour is primarily motivated to solve the coincidence problem - any observers occurring during late times will observe $r \approx 0.35$.

2.3. K-Essence and Interacting Quintessence

⁴ The tracker paths are attractor solutions of the equations governing the evolution of the field. If the field is initially endowed with another density (e.g. an equipartition of the energy available at reheating) the density quickly approaches and joins the tracker solution.

In k-essence the DE is modeled as a scalar field with non-canonical kinetic energy (Chiba et al. 2000; Armendariz-Picon et al. 2000, 2001; Malquarti et al. 2003). Non-canonical kinetic terms can arise in the effective action of fields in string and supergravity theories. A typical k-essence energy density history is plotted in Figure 2 (the purple line). During radiation domination the k-essence field tracks radiation sub-dominantly (with $w_{de} = w_r (= 1/3)$) as some of the other models in Figure 2. However, for dynamic reasons, no stable tracker solution exists for $w_{de} = w_m (= 0)$. Thus after radiation-matter equality, rather than tracking matter, the k-essence field is driven to an attractor with $w_{de} \approx -1$ (this typically involves a sharp decay in energy of several orders of magnitude).

Interacting quintessence models (Amendola 2000; Zimdahl et al. 2001; Amendola and Quercellini 2003; Guo and Zhang 2005) also avoid fine-tuning by using radiation-matter equality to prompt the transition from a scaling behaviour to a Λ -like state. The field energy decays in a tracking solution during radiation domination. Then, at the start of matter domination, interactions between the quintessence field and matter fields transfer kinetic energy out of the DE freezing it (with $w_{de} \approx -1$). Eventually the matter density decays below the DE and the universe begins accelerating. Interacting quintessence models can be phenomenologically similar to k-essence models, so we have not given an independent example in figure 2.

K-essence and interacting quintessence DDE can dominate only recently without fine-tuning because these models use the matter-radiation equality to prompt the transition to a Λ -like state.

2.4. Phantom Dark Energy

Current observations mildly favor a dark energy equation of state $w_{de} < -1$ (peek into section 3). These values are unattainable by standard quintessence models but can occur in phantom field models (Caldwell 2002), which have negative kinetic energies. The energy density in the phantom field *increases* with scalefactor typically leading to a future “big rip” singularity where the scalefactor becomes infinite in finite time. Caldwell et al. (2003) and Scherrer (2005) have explored how big rips may solve the coincidence problem: since such cosmologies spend a significant fraction of their lifetimes in coincidental ($r \geq 0.1$) states it may seem reasonable to expect to observe a near-coincidence. The orange line in Figure 2 illustrates the density history of a simple phantom model with a constant equation of state $w = -1.25$.

2.5. Chaplygin Gas

A special fluid known as Chaplygin gas motivated by braneworld cosmology may be able to play the role of dark matter *and* the dark energy (Bento et al. 2002; Kamenshchik et al. 2001). Generalized Chaplygin gas has a simple equation of state $p_{de} = -A\rho_{de}^{-\alpha}$ which behaves like pressureless dark matter $w_{de} = 0$ at early times, and like vacuum energy $w_{de} = -1$ at late times. The observed dark energy density and equation of state can be matched by fine-tuning the free parameter A . In Figure 2 we show an example (the green line) with $\alpha = 2$ and $A = blarg$ (tuned to match observations).

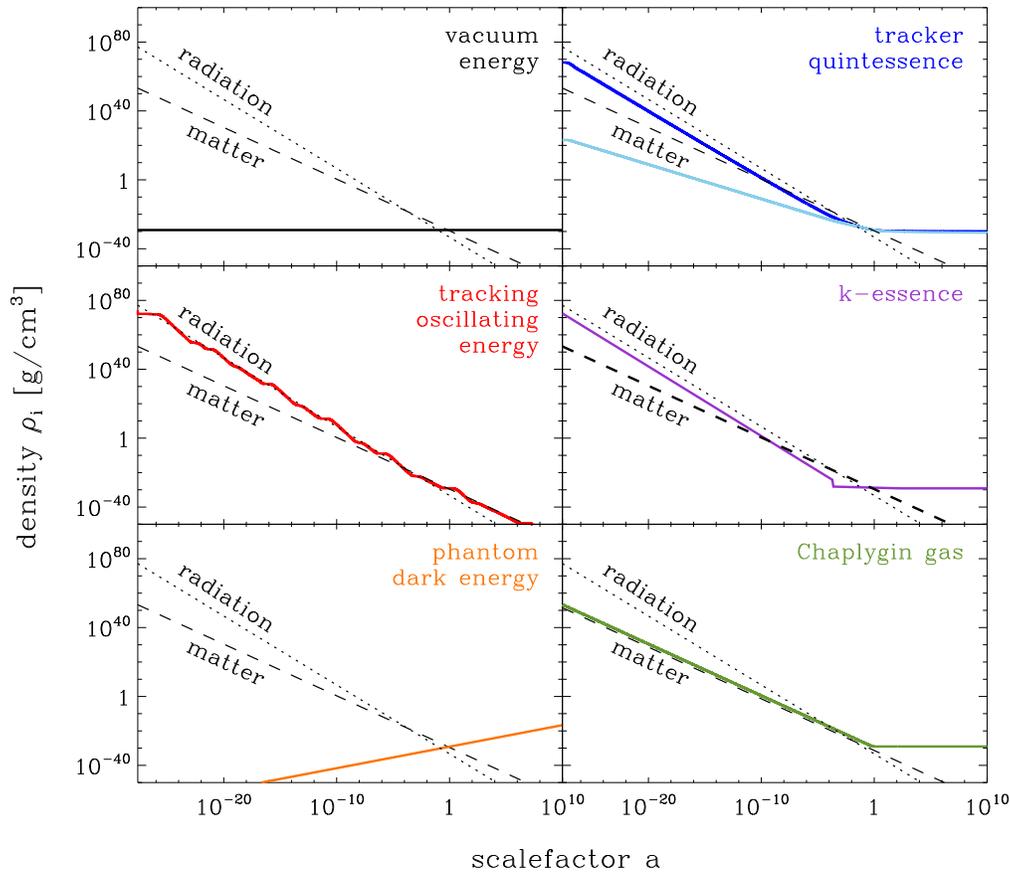


FIG. 2.— The energy density history of the universe according to Λ CDM, and six DDE models selected from the literature. Of these DDE examples, tracking quintessence, k-essence and chaplygin gas models shown here have $\rho_\Lambda \sim \rho_m$ for just a small fraction of the lifetime of the universe. On the other hand, tracking oscillating energy and phantom DE exhibit $\rho_\Lambda \sim \rho_m$ for large fractions of the lifetime of the universe. Just when, and for how long must $\rho_\Lambda \sim \rho_m$ in order to solve the cosmic coincidence?

2.6. Summary

A dichotomy of attitudes towards the coincidence problem is clear in these examples:

1. Λ CDM, tracker quintessence, K-essence, some interacting quintessence and Chaplygin gas models can avoid the coincidence problem if our “time of observation” t_0 has been selected from a narrow and well-timed distribution.
2. Tracking oscillating energy, some phantom models and late time tracking models may solve the coincidence for temporally extended distributions for t_0 (e.g. constant over the lifetime of the universe in *time* or $\log(\text{time})$ - see discussions in Lineweaver and Egan 2007).

The importance of an estimate of the distribution for t_0 is highlighted: such an estimate will either rule out the models of the first category, or demotivate models of the second.

3. CURRENT OBSERVATIONAL CONSTRAINTS ON DYNAMIC DARK ENERGY

Observationally, dark energy dynamics is inferred almost solely from measurements of the cosmic expansion history. Recent cosmic expansion is most directly probed

by using type 1a supernova (SN1a) as standard candles (Riess et al. 1998; Perlmutter et al. 1999). Contemporary data-sets (Riess et al. 2007; Wood-Vasey et al. 2007) include supernova from redshifts $z \lesssim 1.8$ ($a \gtrsim 0.36$).

The position of the first acoustic peak in the CMB temperature angular power spectrum tightly constrains the scale of the universe at last scattering, $z \approx 1089$ ($a \approx 0.001$) (REFS).

Galaxy surveys of the late universe reveal baryonic acoustic oscillations (BAO) in the power spectrum of matter density fluctuations on scales corresponding to the sound horizon at last scattering. Thus BAO measurements tightly constrain the amount of expansion that has gone on between last scattering and the present (Eisenstein and Hu 1998; Blake and Glazebrook 2003; Eisenstein et al. 2005).

Estimated ages of the oldest known white dwarfs (Hansen et al. 2002; Feng et al. 2005) set a lower limit on the current age of the universe - ruling out dark energy models which predict the universe to be any younger than about $t_{wd \min} = 12 \text{ Gyrs}$. Other objects can also be used to set this age limit (e.g. Frebel et al. 2007), but generally less successfully due to uncertainties in dating techniques. See also Lineweaver (1999).

In addition to the constraints on the expansion history (SN1a, CMB, BAO and $t_{wd \min}$) we know that

$\rho_{de}/\rho_{tot} < 0.045$ during Big Bang Nucleosynthesis (Bean et al. 2001). Larger dark energy densities would imply an increased expansion rate at that epoch ($z \sim 10^{10}$) resulting in lower neutron to proton ratio, conflicting with measured abundances.

Because of the variety in which DDE models come, it has become usual to parameterize DE dynamics by expanding the equation of state to 2nd order around the current epoch: $w(a) = w_0 + w_a(1 - a)$. This parameterization is certainly suitable until at least the earliest SN1a, and perhaps as early as last scattering. If one assumes this parameterization is suitable until last scattering, then all cosmological probes can be combined to constrain w_0 and w_a . In a recent analysis of SN1a, CMB and BAO observations Davis et al. (2007) found $w_0 = -1.0 \pm 0.4$ and $w_a = -0.4 \pm 1.8$ (2σ confidence). Using the same observations Wood-Vasey et al. (2007) assumed $w_a = 0$ and found $w = -1.09 \pm 0.16$.

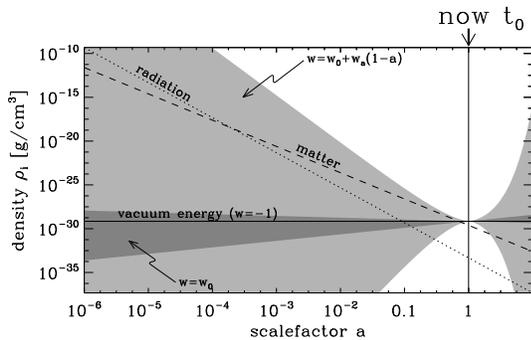


FIG. 3.— The energy densities of radiation ρ_r , matter ρ_m and the cosmological constant ρ_Λ are shown as a function of scalefactor, by the dotted, dashed, and solid lines respectively. Observationally allowed DDE models are shown with grey bands: the light band envelopes models with $w_0 = -1.0 \pm 0.4$ and $w_a = -0.4 \pm 1.8$ and the dark band envelopes models with $w = -1.09 \pm 0.16$ ($w_a = 0$ assumed). DDE models, including examples given in the previous section, have exploit observational uncertainties in the DE density history to allow $\rho_{de} \sim \rho_m$ for extended periods thus removing the apparent coincidence.

The cosmic energy density history is illustrated in Figure 3. Radiation and matter densities steadily decline as the dashed and dotted lines. With the DE equation of state parameterized as $w(a) = w_0 + w_a(1 - a)$, its density history is constrained to the light-grey band (Davis et al. 2007). If evolution of w is negligible then $w(a) = w_0$ and its density history lies within the dark-grey band (Wood-Vasey et al. 2007). If the dark energy is pure vacuum energy (or Einstein’s cosmological constant) then $w = -1$ and its density history is given by the horizontal solid black line.

4. OBSERVATIONAL SELECTION EFFECTS

Brandon Carter introduced the Anthropic Principle (AP) at the IAU symposium in 1973.

The Anthropic Principle: “*What we can expect to observe must be restricted by the conditions necessary for our presence as observers*”. - Carter 1974

Robert Dicke had used the principle to explain away Paul Dirac’s large-number coincidences (Dirac 1938; Dicke

1961) and had encouraged Carter to develop and publish the principle.

The AP was intended as a simple statement about observational selection effects (Carter 2006), and as such must not be ignored when applicable. Consider the following trivial example. Based on the properties of our Earthly environment (selected for ease of measurement), one could argue that the universe has a baryonic density of $\sim 1gcm^{-3}$. Of course this is not true. The average baryonic density is $10^{-31}gcm^{-3}$. Just 10^{-40} of the universe (by volume) is as dense as the Earth. We necessarily observe such a rare location because high densities are required for complicated chemistry and life. The AP reduces the significance of our Earthly observation in determining the properties of the universe.

We are not trying to explain the values of fundamental constants nor promoting multiverse models. We will only be using the AP to estimate the times (and density ratios measured by r) which we might reasonably expect to have observed in each model.

We want to be clear about the following philosophical points which are typically brushed-over. (1) What is the class of observers to which we imagine we belong and (2) in what sense are we to suppose we are typical amongst them? These affect even the most trivial anthropic arguments - including the example about determining the density of the universe (see e.g. Bostrom 2002). Our assumptions are as follows.

1. Reference class: Our reference class is the set of sentient observers belonging to the cosmology-discovering generation of their civilization, the “first observers”. Advanced generations are not counted in the reference class, since they have an intimate understanding of the dark energy and don’t ask about the significance of the observed values of ρ_{de} and ρ_m . That we find ourselves amongst the first observers of our civilization is of debated significance in other respects (see the doomsday argument in e.g. Bostrom 2002). By reducing the reference class to the set of first observers we have isolated the current analysis from speculations about civilization lifetimes. Of course, if one were to take the doomsday argument seriously one would conclude that civilization lifetimes are typically short, and our $P_t(t)$ would correspond approximately with the temporal distribution of all observers, not just first observers.
2. Self-sampling: We will reason as though we were randomly selected from the set of all observers in our reference class. Thus small probabilities are associated with models in which our cosmic observations (such as the observed coincidence $\rho_m \sim \rho_{de}$) are anomalous compared to those of other members of our reference class.

In the following section we estimate the temporal distribution of members of our reference class, as this will allow us to quantify correlations between observations and observational outcomes.

5. ESTIMATING THE TEMPORAL DISTRIBUTION OF FIRST OBSERVERS

The most abundant elements in the cosmos are hydrogen, helium, carbon and oxygen (REF). In the past decade > 200 extra solar planets have been observed via doppler or transit methods and though observational techniques have limited detection to “hot Jupiters”, the findings are consistent with the idea that planetary systems like our own are common in the universe. These do not necessarily imply that observers are common, but they do provide good reasons for believing that terrestrial-planet-bound carbon-based observers are the *most* common.

Lineweaver (2001) estimated the terrestrial planet formation rate (PFR) by making a compilation of measurements of the cosmic SFR (optical for low redshift, radio for high redshift) and suppressing a fraction of the early stars $f(t)$ to correct for the fact that the metallicity was too low for those early stars to host planetary systems.

$$PFR(t) = const \times SFR(t) \times f(t) \quad (2)$$

In Figure 4 we plot the PFR reported by Lineweaver (2001) as a function of redshift, $z = 1/a - 1$. As illustrated in the figure, there is large uncertainty in the normalization of the formation history. Our analysis will not depend on the normalization of this function so these uncertainties *will not* propagate into our analysis. There are also uncertainties in the location of the turnover at high redshift, and in the slope of the formation history at low redshift - both of these *will* affect our results.

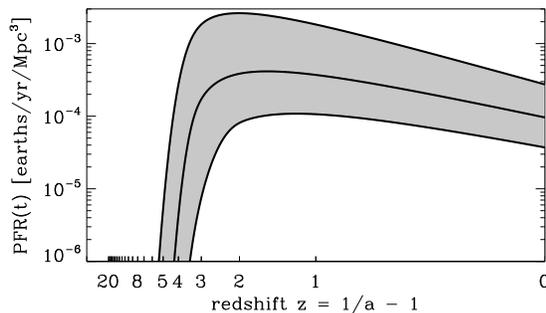


FIG. 4.— The terrestrial planet formation history as estimated by Lineweaver (2001). It is based on a compilation of SFR measurements and has been corrected for the low metallicity of the early universe (which prevents the terrestrial planet formation rate to rise as quickly as the stellar formation rate at $z \gtrsim 4$).

The conversion from redshift to time depends on the particular cosmology, through the following Friedmann equation (which has been generalized to include parameterized dynamic dark energy).

$$\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3} \left[\rho_{r0} a^{-2} + \rho_{m0} a^{-1} + \rho_{de0} \exp[3w_a(a-1)] a^{-3[w_0+w_a+1]} \right] \quad (3)$$

In Figure 5 we plot the PFR from Figure 4 as a function of time assuming the best fit parameterized DDE cosmology. We have re-normalized them to highlight the sources of uncertainty important for this analysis: uncertainty in the width of the function, and in the location of its peak.

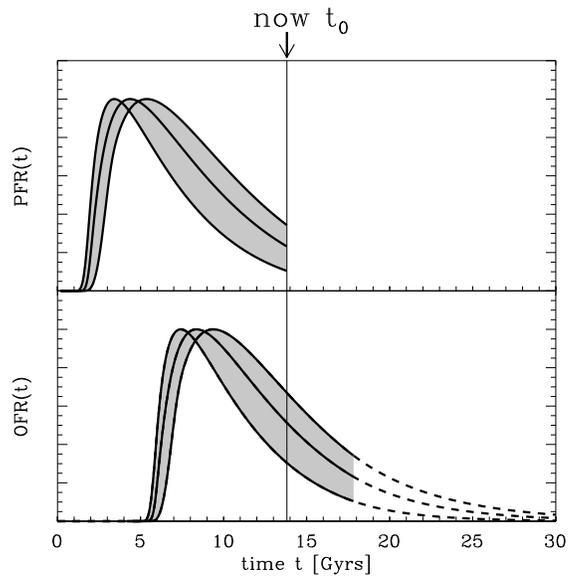


FIG. 5.— The terrestrial planet formation from figure 4 is shown here as a function of time. The transformation from redshift to time is cosmology dependent. To create this figure we have used best-fit values for the DDE parameters, $w_0 = -0.4$ and $w_a = -1.0$. The y-axis is linear (c.f. the logarithmic axis in figure 4) and the family of curves have been re-normalized to highlight the sources of uncertainty important for this analysis: uncertainty in the width of the function, and in the location of its peak.

The observer formation history is calculated by shifting the planet formation history by some amount Δt_{obs} ($= 4Gyrs$ fiducially) to allow for the planet to cool, biogenesis to occur, and a scientific civilization to emerge. These distributions are closed by extrapolating exponentially in t . Physically, this decline in planet formation or observer formation is due to the slowing of various processes: reduced galaxy-galaxy collisions as the universe expands, and depleting local hydrogen stores from which to form stars.

After a star has formed, some non-trivial amount of time Δt_{obs} must pass before observers arise on an orbiting planet. This time allows the planet too cool, biogenesis to occur, and a scientific civilization to emerge via evolution. Owing to the chaotic nature of each of these processes Δt_{obs} could be described by a probability distribution $P_{\Delta t_{obs}}(t)$. The observer formation rate (OFR) would then be given by the convolution

$$OFR(t) = const \times \int_{-\infty}^{\infty} PFR(\tau) P_{\Delta t_{obs}}(t - \tau) d\tau. \quad (4)$$

In practice we know very little about $P_{\Delta t_{obs}}(t)$. It must be very nearly zero below about $\Delta t_{obs} \sim 0.5 Gyrs$ - this is the amount of time it takes for terrestrial planets to cool and the bombardment rate to slow down. Also, it must be near zero above about $\Delta t_{obs} \sim 500 Gyrs$ - the lifetime of a modestly sized star. $P_{\Delta t_{obs}}$ must also have significant weight around $\Delta t_{obs} = 4 Gyrs$ - this is the location of our lone data point, the amount of time it has taken for us to evolve here on Earth. Since the effect of a wider $P_{\Delta t_{obs}}$ is to exacerbate the coincidence problem, we will use the narrowest (most conservative) form we can $P_{\Delta t_{obs}}(t) = \delta(t - 4 Gyrs)$. We will return to discuss this choice retrospectively in section 7. With this choice for $P_{\Delta t_{obs}}(t)$ equation 4 becomes

$$OFR(t) = const \times PFR(t - 4 Gyrs), \quad (5)$$

and the OFR is merely the PFR shifted to the future by 4 Gyrs. This is plotted in the lower panel of figure 5.

The OFR is then closed by extrapolating with a decaying exponential (the dashed segment in figure 5). Physically, the decaying SFH rate is due to a combination of reduced galaxy-galaxy collisions, reduced accretion of gasses into galaxies, and galaxies running out of local hydrogen with which to form stars. Current numerical simulations indicate that all these factors are significant although difficulties in modeling stellar formation and baryon-feedback mechanisms prevent us from saying conclusively. The observed SFH is consistent with a decaying exponential during late times. We have tested other choices (linear & polynomial decay) and our results don't depend strongly on the shape of the extrapolating function used. The exponential decay is done in time rather than z or a - this choice is natural since the future stellar formation rate will more likely depend on the evolution of processes inside virialized spiral galaxies and therefore be largely independent of the increasing cosmic scalefactor a .

The temporal distribution of first observers $P_t(t)$ is proportional to the observer formation rate.

$$P_t(t) = \text{const} \times \text{OFR}(t) \quad (6)$$

Some of our cosmologies suffer big-rip singularities in the future. In these cases we truncate $P_t(t)$ at the big-rip.

6. DOES FITTING CONTEMPORARY CONSTRAINTS NECESSARILY SOLVE THE COSMIC COINCIDENCE?

For a given model the proximity parameter observed by a randomly selected member of our reference class (outlined in the previous section) is described by a probability distribution $P_r(r)$ calculated as

$$P_r(r) = \sum \frac{dt}{dr} P_t(t(r)). \quad (7)$$

The \sum here is the summation over contributions from the multiple solutions of $t(r)$. P_r is shown for the $w_0 = -1.0$, $w_a = -0.4$ cosmology in figure 6.

The ‘‘severity’’ of the cosmic coincidence problem can be defined as the likelihood that a randomly selected member of our reference class observes a proximity r lower than we do.

$$S = P(r < r_0) = 1 - P(r > r_0) = \int_{r=0}^{r_0} P_r(r) \quad (8)$$

If the severity of the cosmic coincidence is near 0.99 (0.99) in a particular model, then that model suffers a 2σ (3σ) coincidence problem: the value of r we observe really is unexpectedly high. For the $w_0 = -1.0$, $w_a = 0.4$ cosmology of figures 5 and 6 the $P_r(r)$ distribution is integrated to give a severity $S = 0.33 \pm 0.07$. Clearly, this model does not suffer a coincidence problem.

We calculated the severities S for cosmologies spanning a large region of the $w_0 - w_a$ plane and show our results in figure 7 using contours of equal S .

We find that *all* observationally allowed combinations of w_0 and w_a result in low severities ($S < 0.6$). I.e. there are large ($> 40\%$) likelihoods of observing the matter and vacuum density to be at least as close as we do. The values for S in the observationally allowed region are given with their uncertainties in table 1.

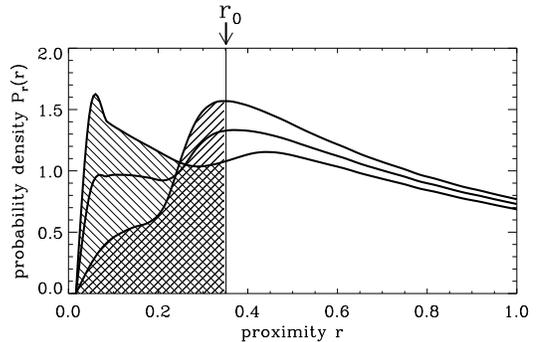


FIG. 6.— The predicted distribution of observations of r is plotted for the parameterized DDE model which best-fits cosmological observations: $w_0 = -1.0$ and $w_a = -0.4$. The value we observe $r_0 = 0.35$ is completely mediocre in this cosmology, as evidenced by this distribution, with at most 40% of observers in our reference class observing $r > 0.35$ (thin striped area) and at least 26% (thick striped area). Thus the ‘‘severity’’ of the cosmic coincidence in this model is $S = 0.33 \pm 0.07$.

TABLE 1
SEVERITIES IN THE OBSERVATIONALLY ALLOWED REGION

	$w_0 = -1.4$	$w_0 = -1.0$	$w_0 = -0.6$
$w_a = -2.2$	0.60 ± 0.03	0.49 ± 0.02	0.31 ± 0.02
$w_a = -0.4$	0.50 ± 0.01	0.33 ± 0.07	0.30 ± 0.11
$w_a = 1.4$	0.26 ± 0.08	0.29 ± 0.10	0.36 ± 0.11

These are the ‘‘severities’’ $S = P(r < r_0)$ of several parameterized DDE cosmologies spanning a coarse grid of points in the observationally allowed region of the $w_0 - w_a$ plane. All w_0 and w_a combinations in this region are acceptable at the 1σ level ($S < 68\%$).

7. CONCLUSIONS

Prior to this analysis it was not clear what DDE dynamics were required to solve the coincidence problem. In the introduction we noted the possibility that these investigations might have resulted in new constraints on the values of w_0 and w_a , by simply demanding that we do not live during a special time in which $\rho_{de0}/\rho_{m0} \sim 1$ (while potential members of our reference class observe ρ_{de0}/ρ_{m0} to be vastly different from 1). Excluding models based on a-prosteriori statistical tests is dangerous. It may be the case that we have chosen to analyze the ρ_m/ρ_{de} coincidence *because* those densities are unexpectedly close. For example, the ratio ρ_r/ρ_{de} is equally significant, but the coincidence problem was not defined in terms of those densities because their ratio is much smaller (and less surprising). Nevertheless, the ratio ρ_m/ρ_{de} is a rather obvious/fundamental quantity and we can have selected it from only a handful such quantities (ρ_m/ρ_{de} , ρ_r/ρ_{de} and perhaps a few other cosmologically significant ratios). If we assume the cosmic coincidence (as defined) is the most strong of $O(5)$ potential tests, then we can account for its a-prosteriori selection by demanding $S > 0.99$ before ruling the model out with 95% confidence. There are regions of $w_0 - w_a$ parameter space that can be ruled out in this manner (see figure 7) however those points are already strongly excluded by observations. Therefore, as it turns out, the coincidence problem is too mild and we cannot extract new constraints on w_0 and w_a in this way.

Instead the result of this paper is that any DDE model in agreement with current cosmological constraints has

$\rho_{de} \sim \rho_m$ for a significant fraction of observers in the relevant reference class.

XXX What does Roberto Trotta do?

XXX Hebecker and Wetterich (2001) asks about ‘‘Natural Quintessence?’’

XXX Somebody 2007 wrote a paper explaining that we see taking over now because if it were different we would not be able to see it.

XXX Del Campo et al. (2006); del Campo et al. (2005) tries to ‘‘soften’’ the coincidence.

Caldwell et al. (2003) and Scherrer (2005) have proposed that the coincidence problem may be solved by phantom models in which there is a future big-rip singularity because such cosmologies spend a significant fraction of their lifetimes in coincidental ($r \geq 0.1$) states. In our work $P_t(t)$ is terminated by big-rip singularities in ripping models. In non-ripping models, however, the distribution is effectively terminated by the declining star formation rate. Therefore we argue that the big-rip provides phantom models only a marginal advantage over other models. This marginal advantage manifests as the discontinuity along $w_a = 0$ in the left of figure 7.

The primary weakness of this analysis was in modeling $P_{\Delta t_{obs}}$. We had to use the most conservative possibility - a delta function. A less conservative choice would yield larger severities than those in figure 7 but we feel that

it’s unlikely that any well-motivated deviations from our choice $P_{\Delta t_{obs}}$ would change our main result. Another weaknesses is the DE equation of state parameterization used ($w = w_0 + w_a(1 - a)$), which may not parameterize some models well.

We conclude that DDE models need not be fitted with exact tracking or oscillatory behaviors specifically to solve the coincidence by generating long or repeated periods of $\rho_{de} \sim \rho_m$. Moreover phantom models have no significant advantage over other DDE models with respect to the coincidence problem discussed here.

Fitting current cosmological constraints naturally with DDE models is a non-trivial task. Some form of tuning in the Lagrangian is required: tuning of the potential $V(\phi)$, or ad-hoc kinetic terms or interactions (see the examples in section 2). We want to re-iterate that we do not deal with these ‘‘fine tuning’’ problems, and the use of tracking and oscillatory behaviours may still be helpful in avoiding such tuning. Nevertheless our result, which has already been explicitly assumed in several of the most successful and dynamically simple DDE models reduces the number of considerations that need to go into future DDE models.

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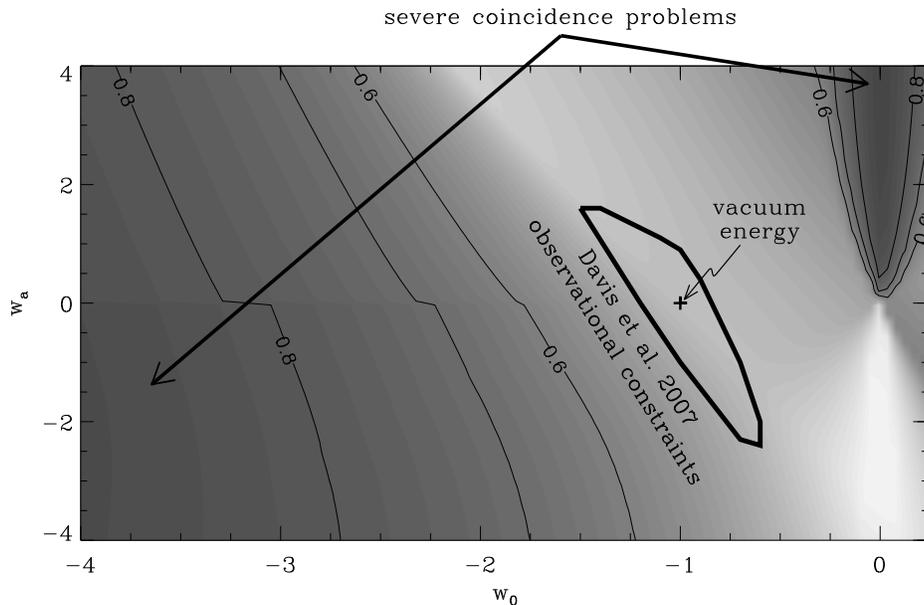


FIG. 7.— Here we plot contours of equal “severity” S in $w_0 - w_a$ parameter space. S is the fraction of observers who see $r < r_0$. The thick black contour represents the observational constraints on w_0 and w_a from Davis et al. (2007) (2σ confidence and marginalized over other uncertainties). In (Lineweaver and Egan 2007) we showed that the severity of the coincidence problem is low for Λ CDM (indicated by the “+”). This plot shows that regions of parameter space that can be ruled out by the cosmic coincidence are already strongly excluded by observations. This plot contains our main result: any DDE model fitting current observational constraints on w_0 and w_a does not suffer a coincidence problem. Some features are worth noting:

- (1) Dominating the left of the plot, as w_0 and w_a become more negative, there is a general increase in severity of the coincidence. This is because r peaks more recently and for a briefer period of time.
- (2) A strong vertical dipole of coincidence severity centered at $(w_0 = 0, w_a = 0)$. For $(w_0 \approx 0, w_a > 0)$ there is a large coincidence problem because in such models we would be currently witnessing the very closest approach between DE and matter, with $\rho_{de} \gg \rho_m$ for all earlier and later times. For $(w_0 \approx 0, w_a < 0)$ there is an anti-coincidence problem because in those models we would be currently witnessing the DDE’s very furthest excursion from the matter density, with ρ_{de} and ρ_m in closer proximity for all relevant earlier and later times.
- (3) There is a sharp discontinuity in the contours running along $w_a = 0$ for phantom models ($w_0 < -1$). The first-observer distribution $P_t(t)$ is truncated by big-rip singularities in strongly phantom models (provided they remain phantom; $w_a > 0$). This truncation of late-time observers means that early observers who witness large values of r represent a greater fraction of the total population.

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