Problem 1: (5pts)
Show that, for a matter dominated Universe, with no cosmological constant, no photons, and nothing else of interest, the k=-1 solution of the Friedmann equations is given by:

$$\frac{a}{a_0} = \frac{\Omega_0}{2(1-\Omega_0)}(\cosh\eta - 1)$$

$$t = \frac{1}{H_0}\frac{\Omega_0}{2(1-\Omega_0)^{3/2}}(\sinh\eta - \eta)$$

Problem 2: (3pts)
Assuming a matter-dominated Universe (as above) and a value of the Hubble constant of 70 km/s/Mpc, calculate the age of the Universe in Billions of years for 3 values of the density parameter, Ω_M=1,0.3,1.3.

Problem 3: (5pts)
Show that at early times (t small), the k=-1,k=0, and k=+1 solutions of the matter dominated Friedmann equations are indistinguishable.

Problem 4: (2pts)
The current temperature of the Universe is 2.73 degrees K. Given that Blackbody radiation has an energy density given by the expression $\rho_{rad} = \frac{4}{c}\sigma T^4$, where σ is the Stefan Boltzmann Constant, compute the current energy density in radiation and compare this to the critical density

Problem 5: (2pts)
Assuming that the current matter density of the Universe is Ω_M =0.27 and H_0=70 km/s/Mpc, compute the redshift where Ω_M and Ω_{rad} are equal.

Problem 6: (2pts)
Assuming radiation is blackbody (obeys Planck's law), show that the temperature of the Universe scales as $T \propto (1+z)$

Problem 7: (1 pt)
Using your results from above, what was the temperature of the Universe when radiation and matter made equal contributions to the overall density of the Universe.

Problem 8. On 22nd February 1987, a supernova appeared in the Large Magellanic Cloud. When this object was observed with the

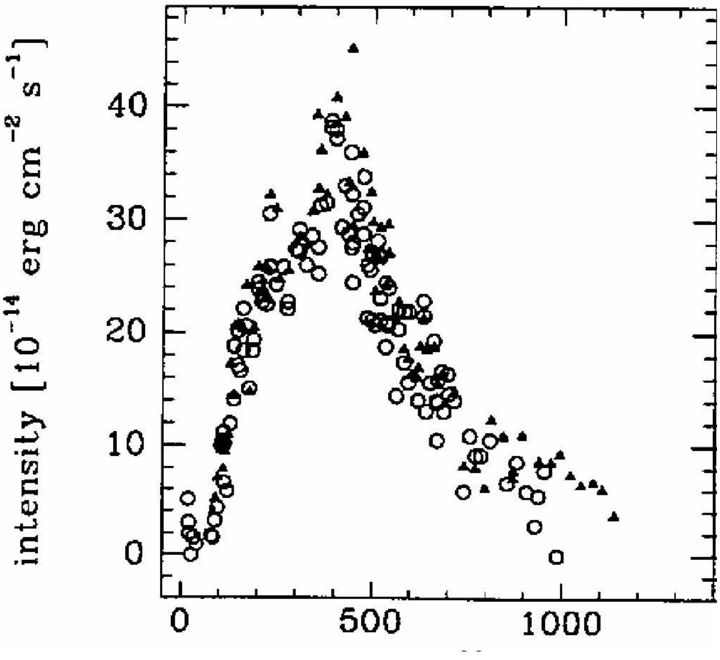

Fig 1: Days since explosion

IUE Ultraviolet Satellite, it initially contained spectral features associated with the supernova, but about 100 days after the explosion, suddenly, very narrow emission lines of Nitrogen appeared and gained in brightness for 300 days, before fading away. See figure 1 for the light curve of these narrow Nitrogen lines. When the Hubble Space Telescope was launched, one of the first pictures it took was of SN 1987A (figure 2), and the situation was revealed. There was a narrow ring of material (+ two additional fainter rings we will ignore here) surrounding the SN
1.66"x1.11" in diameter along the major and minor axis, respectively (Figure 2).

Let's assume that the UV and X-rays from the Supernova, when it first exploded, ionized this ring of gas, and the light curve that we see is then a result of light travel time effects. We can then measure the distance to SN 1987A in a purely geometric way!

8.1 (2 pts) Draw a diagram of the situation where the Earth is a distance *D* from the SN down the y-axis, the ring is spherical and is located at radius *r* from the SN, and is inclined an angle *i* with respect to the x-y plane. The position of the ring is measured by θ, an angle in the plane of the ring, which is at 0 degrees down the x-axis, 90 degrees when projected onto the y-axis. Using measurements from figure 2, what is the inclination angle?

8.2 (5pts) Show that the difference in light travel time from the SN to the Earth versus the SN to the ring to the Earth is given by the expression $\Delta t = (1 - \sin\theta \cos i)\frac{r}{c}$ if r << D.

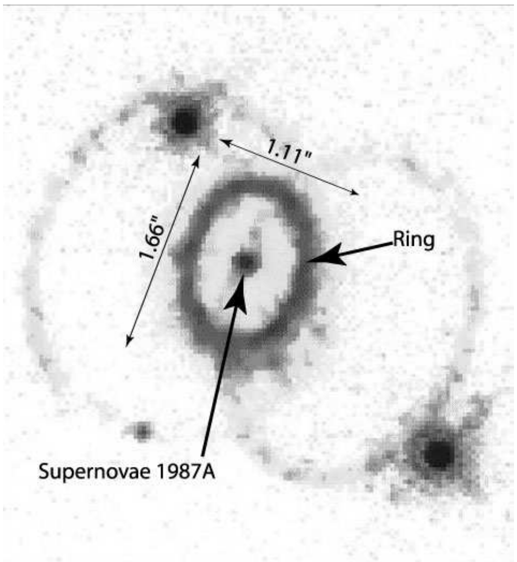

Fig 2: HST Picture of SN 1987A

8.3 (5pts) Each parcel of gas in the ring, when it gets hit by the initial burst of SN light, gets ionized, and then recombines when it encounters a friendly electron. This recombination produces the narrow emission lines of Nitrogen shown in figure 1. If we assume that the chance of a recombination occurring is constant per unit time, what form (equation) will the number of recombinations per unit time take for a group of atoms all ionized at the same time. Use this, in combination with the time delay from 8.2, to model the light curve (an approximate fit to the data is fine) shown in figure 1. Comment on the whether or not you think your modeled light curve is appropriate for the situation.

8.4 (3pts) Using expression from problem 8.2, derive expressions for the time after SN explosion date, when the first light from the ionized ring reaches Earth. If, from figure 1, the turn-on of the SN occurred at 83 days, measure the physical distance, *r* from the SN to the ring. If the maximum brightness of the ring occurs at the point when light from the whole ring first reaches the Earth, derive the physical distance, *r* from the SN to the ring, assuming this happens 380 days after the SN explosion. Using the average of the two values you have derived for *r*, and the angular size of the ring, calculate the distance to SN 1987A. In measuring their Hubble Constant of 72 km/s/Mpc, the Key Project assumed that the LMC was located 50kpc from Earth. What Hubble constant would they have gotten if they had used your distance to the LMC?

be subtracted off, to make a measurement. Show that, if the uncertainty in a measurement (signal) of N photons is sqrt(N), and that the signal of the background dominates the signal of the object, the relative time to achieve the same signal/noise (that is signal divided by noise) of two objects of different fluxes is given by the expression. $\frac{t_1}{t_2} = \left(\frac{F_2}{F_1}\right)^2$

9.4 (5pts)

We want to use supernovae to measure luminosity distance as a function of redshift. We have a choice, we can compare our nearby objects at z=0.01 with objects at z=1 or at z=0.5. But we want to use the least amount of telescope time. We will assume that Gaussian statistics apply. That is, the ratio of the number of measurements required to achieve two different signals scales as

$$\frac{N_1}{N_2} = \left(\frac{Signal_1}{Signal_2}\right)^2$$

For example. If I have a method of measuring a person's height which has an uncertainty associated with it (which obeys Gaussian statistics), and I want to double my accuracy, I need to make 4 times more measurements.

Using your results in 9.1-9.4, calculate the ratio of time it will take to discriminate between Universe 1 and Universe 2 comparing nearby objects to objects at z=0.5 or z=1.