Problem Set 2 - Cosmology–A3002 –Due on or before November 7^{th}. Can be sent via internal mail (Brian Schmidt, RSAA)

use H_0=70 km/s/Mpc, Ω_M=0.27, Ω_Λ=0.73, w=-1 for all problems.

Please send my course materials to this Address: (I can just as easily scan and send to an email address)

Problem 1.1: (4pts)
Imagine we want to use supernovae as a standard candle to measure luminosity distance as a function of redshift in order to determine the Hubble Constant. From class we know that selection effects can bias our measurement. We want to simulate the effect using a Monte Carlo simulation. First, lets determine the bias as a function of redshift for a sample of objects via a Monte Carlo calculation. To set up your simulation, assume that H_0 = 70 km/s/Mpc (and ignore effects Ω_M , Ω_Λ) and that the absolute magnitude of SN Ia =-19.5 mag, and that your supernova search will be able to detect a 100% of objects as faint as 18.5 magnitude, and none fainter. Using a computer program or excel spreadsheet, calculate the mean magnitude of the SNIa actually detected at z=0.05, and z=0.09 for 3 assumptions of the dispersion (Gaussian 1 std-deviation) of SNIa about their mean brightness of 0.1, 0.2, and 0.4 magnitudes. An excel hint: A Gaussian random variable with mean -19.5 and std deviation 0.2 in EXCEL is given by NORMINV(RAND(),-19.5,0.2). You are free to use any computer program to do the simulation.

Problem 1.2: (4 pts)
To simulate the bias in measuring the Hubble Constant, we need to realistically select objects as a function of redshift. In the nearby Universe, we can assume that volume is proportional to z^3. If we want to randomly select objects in the volume between $0<z<z_{max}$, the selection of a random redshift is given as

$$z(\text{random}) = z_{max}\,(\text{RANDOM})^{1/3}$$

where RANDOM is a random number between 0 and 1.

Finish off your Monte Carlo simulation by selecting objects at random redshifts from 0<z<0.1, assuming that they have a mean absolute magnitude of M=-19.5 with a dispersion 0.2 magnitudes. Using only those objects which appear brighter than your limiting magnitude of 18.5, calculate the Hubble constant from your sample for all visible objects. How does this compare to your input value of 70 km/s/Mpc? Do this for dispersions for SN Ia of 0.3 and 0.5 magnitudes as well and compare the resulting biases in the Hubble Constant.

Problem 2.1 (2 pt):
In class we described the scale at which material can collapse in the Universe as the Jeans' length. This scale is given approximately by the equation

$$L_{Jeans} \approx \sqrt{\frac{kT}{mG\rho}}.$$

Where the mass per particle is given as ***m***, ***T*** is temperature, ρ is density, and ***G*** and ***k*** are the gravitational and Stefan-Boltzmann constants, respectively. As the Universe expanded, a special time occurred when the Hydrogen and electrons recombined – photons and matter were no longer coupled, maybe allowing objects to freely gravitationally collapse. Given that the observed temperature of the CMB is 2.73K, and hydrogen recombines at T=3000K, approximately what redshift did this decoupling occur? What was the density of matter at that time?

Problem 2.2 (2 pt):
Determine the Jeans' length for this epoch? What is the Jeans' mass – i.e. the amount of mass within this scale? As the Universe evolves, how would this size and mass look today?

Problem 2.3 (2 pt):
Globular clusters have been proposed as the first objects to collapse in the Universe. Do some research and suggest some good and bad aspects of the idea that globular clusters are the result of a Jeans' length collapse immediately after recombination?

Problem 3.
Use H_0=70km/s/Mpc, and that the Universe is flat for all of question 3.

Problem 3.1 (4 pts):
The CMB was emitted when the Universe recombined at a Temperature of about T=3000 K. Using the redshift of determined in 2.1, how old was the Universe at that time? You should assume a flat Universe, and that the transition between radiation and matter domination occurred at z=5300 (from last problem set), and that you can break the Universe up into two pieces above and below this transition redshift where it is radiation dominated, and matter dominated.

Problem 3.2 (2 pts):
The sound speed of a material c_s is given by the expression

$$c_s = \sqrt{\frac{\partial p}{\partial \rho}}$$

We have seen from class that the equation of state for photons is given by the expression

$$w_\gamma = \frac{1}{3} = \frac{p}{\rho c^2}$$

Therefore, what is the speed of sound in the photon-dominated early Universe?

Problem 3.3 (5 pts):

Inhomogeneities that eventually will give rise to galaxies and large scale structure, will have a characteristic acoustic scale equal to the speed of sound times the age of the Universe. How big, in kpc, is this scale?

How big should this scale appear on the sky now when we look back at the CMB – do this by calculating the angular-size distance from either the excel spread sheet on the web page, or with your own program, from z=0 to the CMB redshift.

This acoustic scale should be preserved in the galaxies today. How big –e.g. what is the co-moving size - is this acoustic scale at z=0, in Mpc?