Pop Quiz / Collaborative Learning

- You can ask me to show any page from any lecture. Although everyone else get to see it too.
- First Group in with a correct answer gets…

Move to observable space…

$$a(t) = a_0\left(1 + \frac{da}{dt}(t_0)\frac{(t-t_0)}{a_0} - \frac{1}{2}\frac{d^2a}{dt^2}(t_0)\frac{(t-t_0)^2}{a_0^2} + \ldots\right)$$ Taylor Expansion

$$q_0 \equiv -\frac{\frac{d^2a}{dt^2}(t_0)a_0}{\frac{da}{dt}(t_0)^2}$$ Deceleration Parameter.

$$H_0 \equiv \frac{\frac{da}{dt}(t_0)}{a_0}$$ Hubble Constant

$$a(t) = a_0\left[1 + H_0(t-t_0) - \frac{1}{2}q_0 H_0{}^2(t-t_0)^2 + \ldots\right]$$

Substitute in deceleration and Hubble Constants

$$z = H_0(t_0 - t) + \left(1 + \frac{q_0}{2}\right)H_0{}^2(t_0 - t)^2 + \ldots$$

$$(t_0 - t) = \frac{1}{H_0}\left[z - \left(1 + \frac{q_0}{2}\right)z^2 + \ldots\right]$$

Non-trivial algebra…

$$z = H_0(t_0 - t) + \left(1 + \frac{q_0}{2}\right) H_0{}^2 (t_0 - t)^2 + \ldots$$

$$(t_0 - t) = \frac{1}{H_0}\left[z - \left(1 + \frac{q_0}{2}\right) z^2 + \ldots \right]$$

for a light ray, using R - W metric and $ds^2 = 0$

$$\int_t^{t_0} \frac{cdt}{a} = \int_0^r \frac{dr}{\sqrt{1 - kr^2}} = S(r) = r \text{ for the flat case}$$

Substitute in For z

$$\frac{c}{a_0}\int_t^{t_0} (1+z)dt = \frac{c}{a_0}\int_t^{t_0}\left[1 + H_0(t - t_0) + \left(1 + \frac{q_0}{2}\right) H_0{}^2 (t - t_0)^2 + \ldots\right] dt = S(r) = r$$

$$r = \frac{c}{a_0}\left[(t_0 - t) + \frac{H_0{}^2 (t_0 - t)^2}{2} + \ldots \right]$$

Integrate – note can expand S(r) for sin/sinh case and get Same result

$$r = \frac{c}{a_0 H_0}\left[z - \frac{1}{2}(1 + q_0) z^2 + \ldots \right]$$

Substitute in for t_0-t

$$d_{proper} = \int ds = \int_0^r \left(\frac{a(t)dr}{\sqrt{1-kr^2}} \right) = a(t) \begin{Bmatrix} \arcsin(r) & k=+1 \\ r & k=0 \\ \operatorname{arcsinh}(r) & k=-1 \end{Bmatrix} = a(t)S(r)$$

$$d_{proper} = ar$$

$$a(t) = a_0 \left[1 + H_0(t-t_0) - \frac{1}{2} q_0 H_0{}^2 (t-t_0)^2 + \ldots \right]$$

$$(t_0 - t) = \frac{1}{H_0} \left[z - \left(1 + \frac{q_0}{2} \right) z^2 + \ldots \right]$$

Substitute in for t_0-t in a(t)

$$r = \frac{c}{a_0 H_0} \left[z - \frac{1}{2}(1+q_0) z^2 + \ldots \right]$$

$$d_{proper} = \frac{c}{H_0(1+z)} \left[z + \frac{1}{2}(1-q_0) z^2 + \ldots \right]$$

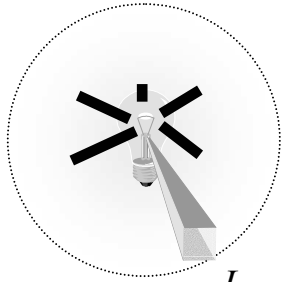
Multiply a(t) and r expansions to get

$$d_L = (1+z) d_{proper} = (1+z)^2 d_A$$

Luminosity Distance – GR + Homogenous Isotropic Universe

for a monochromatic source
(defined as inverse-square law)

$$D_L = \sqrt{\frac{L}{4\pi F}}$$



$$F = \frac{L}{4\pi D^2}$$

the flux an observer sees of an object at redshift z

$$D_L = \frac{c}{H_0}(1+z)\left|\kappa_0\right|^{-1/2} S\left\{\left|\kappa_0\right|^{1/2}\int_0^z dz'\left[\sum_i \Omega_i (1+z')^{3+3w_i} - \kappa_0 (1+z')^2\right]^{-1/2}\right\}$$

$$\kappa_0 = \left(\Omega_{tot} = \sum_i \Omega_i\right) - 1$$

$$S(x) = \begin{cases} \sin(x) & \mathrm{k} = 1 \\ x & \mathrm{k} = 0 \\ \sinh(x) & \mathrm{k} = -1 \end{cases}$$

Brightness of object depends exclusively on what is in the Universe
How much and its equation of state.