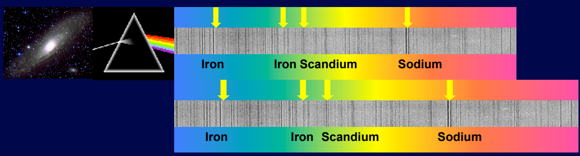
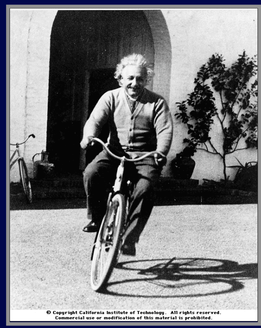
Slipher's Spectra of Nebulae

Lowell Telescope – near Flagstaff Arizona –provided spectra of ~50 nebulae – showing 95% of them were moving away from Earth at speeds up to 1500km/s



General Relativity



Different Model Universes

- 1917 Einstein's Cosmological Constant Universe
- 1916, 1920 de Sitter's Empty Universe Solutions predicted spectra shift
- 1922 Friedmann showed family of solutions based on homogenous isotropic Universe.
- 1930 Einstein de-Sitter Flat Universe
- 1936 Robertson-Walker Solutions

Hubble's Discovery of the Expanding Universe

•1926 Hubble measuring average space density of Galaxies to look at effects of Curvature using static Universe solutions

•1927 Lemaitre showed Hubble Law (velocity proportional to Distance) expected of Friedmann Universes, and demonstrated, using Hubble's/Slipher Data the Law in nature. Noted age of Universe roughly 1/Hubble Constant

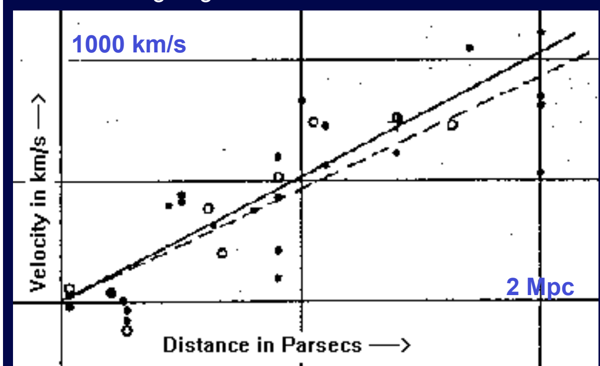
•1928 Robertson predicts linear relationship and claims to see it in plot of Galaxy brightness versus Slipher's redshift

•1929 Humason announces velocity of NGC 7619 of 3779 km/s

•1929 at National Academy of Science Hubble presents paper announcing Universe is Expanding

•1929-1931 Hubble/Humason extend relation to 20000 km/s

Assuming brightest stars are standard Candles

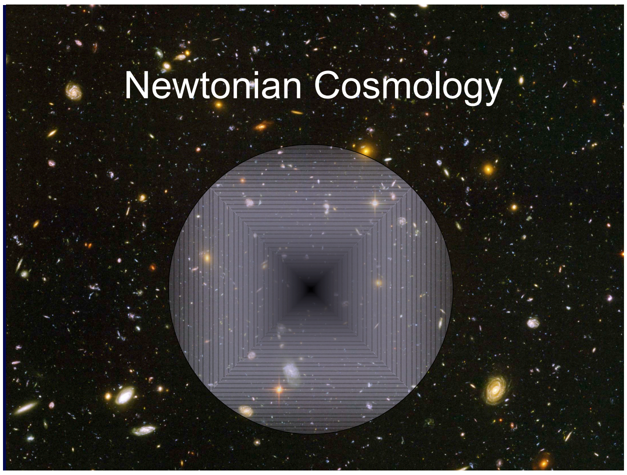


H_0=500 km/s/Mpc = 2 Gyr^{-1}

Lemaitre – The unsung hero

- Fought for Belgium starting at 14 in WWI
- Seminary in 1923, ordained as a priest
- 1923 Visited Eddington in Cambridge England
- 1924 Visited Harlow Shapley in Cambridge, Mass
- 1925 enrolled in PhD at MIT, but returned to Brussels to work on it.
- 1927 published "homogeneous **Universe** of constant mass and growing radius accounting for the radial velocity of extragalactic nebulae"
  - Independently derived Friedman Equations
  - Suggested Universe was expanding
  - Showed it was confirmed by Hubble's data.

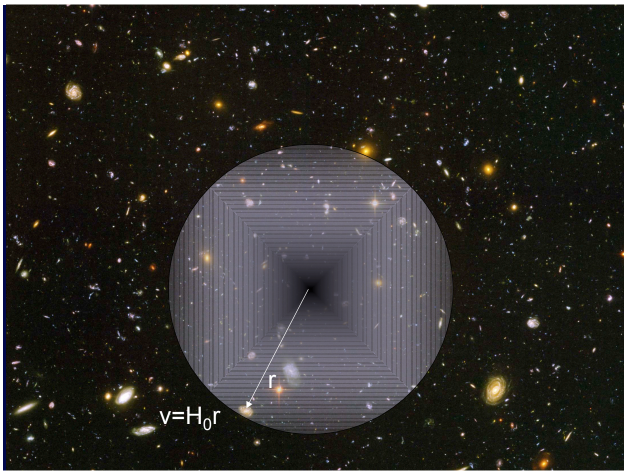
  Mathematics accepted by Einstein, but basic idea rejected by Einstein
- 1931 Discussed primeaval atom which everything grew out of – the Big Bang



Apply Gauss' Law: $\oint G\frac{M}{r^2}\hat{r}\cdot dA = \langle g \rangle A = 4\pi GM$

Since this holds for any sphere, there is a solution for all spheres, M=0

So need to fake it: Lets assume that all forces cancel out, outside the sphere, and we only have to worry about those forces inside the sphere.



- Within shell of radius r,

$$M = \rho_0 \frac{4}{3}\pi r^3$$

  - Note as time moves forward, galaxy moves out, (r increases), but Mass "affecting" object is constant.
  - Equation of Motion for any Galaxy

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \text{constant}$$

Critical Universe

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$\frac{1}{2}(H_0 r)^2 = \frac{G\rho\frac{4}{3}\pi r^3}{r}$$

$$\rho = \frac{3H_0^{\,2}}{8\pi G} \equiv \rho_{crit}$$

Critical Density defines line between bound and unbound Universe. Density higher than this, Universe has negative Energy – Bound, lower than this, Universe has positive Energy - Unbound

The density changes in time, so v=H(t)r

Motion of Critical Universe

$$\frac{dr}{dt} = v$$

$$\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 = \frac{GMm}{r}$$

$$\left(\frac{dr}{dt}\right) = \sqrt{\frac{2GM}{r}}$$

$$r^{1/2}dr = \sqrt{2GM}\,dt$$

$$\int r^{1/2}dr = \int \sqrt{2GM}\,dt$$

$$\frac{2}{3}r^{3/2} = t\sqrt{2GM}$$

$$r \propto t^{2/3}$$

Defined boundary condition

t=0, r=0

Age of Universe:

$$t = t_0$$

$$\sqrt{\frac{2GM}{r}} = \frac{dr}{dt} = v = H_0 r$$

$$H_0 = \sqrt{2GM}\,r^{-3/2}$$

$$r^{3/2} = \frac{\sqrt{2GM}}{H_0}$$

$$\frac{2}{3}\frac{\sqrt{2GM}}{H_0} = \sqrt{2GM}\,t_0$$

$$t_0 = \frac{2}{3}\frac{1}{H_0}$$

General Equation of Motion (non-critical case)

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = K$$

$$\text{at } t_0 \quad K = \frac{1}{2}m(H_0 r_0)^2 - \frac{G\frac{4}{3}\pi r_0^{\,3}\rho_0 m}{r_0}$$

$$K = m\left(\frac{H_0^{\,2}}{2} - \frac{4}{3}\pi G\rho_0\right)r_0^2$$

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{G\left(\frac{4}{3}\pi r_0^{\,3}\rho_0\right)}{r} = \left(\frac{H_0^{\,2}}{2} - \frac{4}{3}\pi G\rho_0\right)r_0^2$$

M is constant as seen by our galaxy, so I have used
the current epoch expression for M for all times

non-critical case, cont

$$D_* = \frac{r}{r_0} \quad \tau_* = H_0 t \quad \Omega_0 = \frac{\rho_0}{\rho_{crit}} = \frac{8\pi G\rho_0}{3H_0^{\,2}}$$

$$\frac{dD_*}{dr} = \frac{1}{r_0} \qquad \frac{dt_*}{d\tau_*} = \frac{1}{H_0}$$

$$\frac{dD_*}{d\tau_*} = \frac{dD_*}{dr}\frac{dr}{dt}\frac{dt}{d\tau_*} = \frac{1}{r_0}\,\frac{dr}{dt}\frac{1}{H_0}$$

$$\frac{dr}{dt} = r_0 H_0 \frac{dD_*}{d\tau_*}$$

Substitution to simplify
This ODE

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{G\left(\frac{4}{3}\pi r_0^{\,3}\rho_0\right)}{r} = \left(\frac{H_0^{\,2}}{2} - \frac{4}{3}\pi G\rho_0\right)r_0^2$$

$$\frac{1}{2}\left(r_0 H_0 \frac{dD_*}{d\tau_*}\right)^2 - G\left(\frac{4}{3}\pi\rho_0 r_0^{\,2}\right)\frac{r_0}{r} = \left(\frac{H_0^{\,2}}{2} - \frac{4}{3}\pi G\rho_0\right)r_0^2$$

$$\left(\frac{dD_*}{d\tau_*}\right)^2 - \frac{8\pi G\rho_0}{3H_0^{\,2}}\frac{r_0}{r} = \left(1 - \frac{8\pi G\rho_0}{3H_0^{\,2}}\right)$$

$$\left(\frac{dD_*}{d\tau_*}\right)^2 - \frac{\Omega_0}{D_*} = (1 - \Omega_0)$$

$$\left(\frac{dD_*}{d\tau_*}\right)^2 - \frac{\Omega_0}{D_*} = (1 - \Omega_0)$$

Again define variables to solve this ODE

$$\xi = \frac{|1-\Omega_0|}{\Omega_0}D_* \qquad \tau = \frac{(|1-\Omega_0|)^{3/2}}{\Omega_0}\tau_*$$

$$\frac{d\xi}{d\tau} = \frac{d\xi}{dD_*}\frac{dD_*}{d\tau_*}\frac{d\tau_*}{d\tau} \qquad \frac{d\xi}{dD_*} = \frac{|1-\Omega_0|}{\Omega_0}\ , \quad \frac{d\tau_*}{d\tau} = \frac{\Omega_0}{(|1-\Omega_0|)^{3/2}}$$

$$\frac{d\xi}{d\tau} = \frac{|1-\Omega_0|}{\Omega_0}\frac{dD_*}{d\tau_*}\frac{\Omega_0}{(|1-\Omega_0|)^{3/2}}$$

$$\frac{dD_*}{d\tau_*} = \frac{d\xi}{d\tau}(|1-\Omega_0|)^{1/2}$$

$$\left(\frac{d\xi}{d\tau}\right)^2 |1-\Omega_0| - \Omega_0\frac{|1-\Omega_0|}{\xi\Omega_0} = (1 - \Omega_0)$$

$$\left(\frac{d\xi}{d\tau}\right)^2 - \frac{1}{\xi} = \frac{(1-\Omega_0)}{|1-\Omega_0|} = \pm 1$$

$$\left(\frac{d\xi}{d\tau}\right)^2 = \frac{1}{\xi} \pm 1$$

$$d\tau = d\xi\sqrt{\frac{\xi}{1\pm\xi}}$$

$$\tau = \int_0^{\xi} d\xi\sqrt{\frac{\xi}{1\pm\xi}}$$

final substitution! for Bound (-) case

$$\xi = \sin^2(\eta/2) = \frac{1}{2}(1-\cos\eta)$$

$$d\xi = \sin(\eta/2)\cos(\eta/2)d\eta$$

$$\tau = \int_0^{\eta} \sin(\eta/2)\cos(\eta/2)d\eta\sqrt{\frac{\sin^2(\eta/2)}{1-\sin^2(\eta/2)}}$$

$$\tau = \int_0^{\eta} \sin(\eta/2)\cos(\eta/2)d\eta\sqrt{\frac{\sin^2(\eta/2)}{\cos^2(\eta/2)}} = \sin^2(\eta/2)d\eta$$

$$\tau = \int_0^{\eta}\left(\frac{1}{2} - \frac{1}{2}\cos\eta\right)d\eta = \left(\frac{1}{2}\eta - \frac{1}{2}\sin\eta\right)$$

Final Parametric Solution

$$\xi = \frac{1}{2}(1-\cos\eta) = \frac{|1-\Omega_0|}{\Omega_0}\frac{r}{r_0}$$

$$\tau = \left(\frac{1}{2}\eta - \frac{1}{2}\sin\eta\right) = \frac{|1-\Omega_0|^{3/2}}{\Omega_0}H_0 t$$