Rotation and Angles

By torque and energy

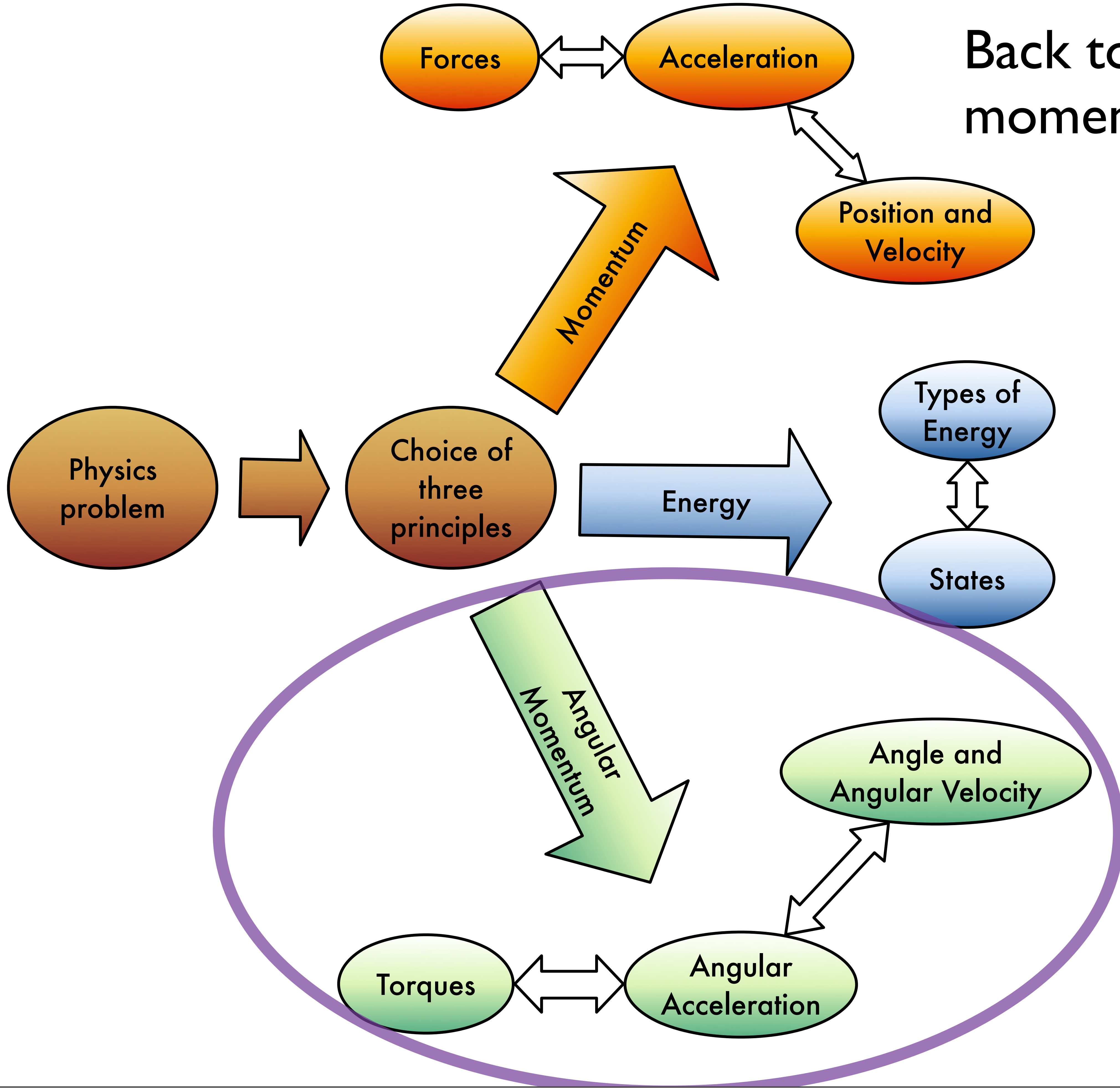
CPR

- An experiment - and things always go wrong when you try experiments the first time. (I won't tell you the horror stories of when I first used clickers, Wattle and Webassign...)
- Due to the various technical glitches, I'll treat the CPR Archer Fish question as a practice run. Marks will be given for participation - as long as you made a decent effort at both writing your work and marking others, you'll get full marks.

Try again

- This week...
- Revised instructions.
- Zoho is fixed and now works well.
- Google Docs are troublesome - avoid or follow new instructions closely.
- It is now your responsibility to check that the web link you post can be read.

Back to angular momentum



Angular momentum so far

- We talked about torques.
- A torque is force times PERPENDICULAR distance.
- If an object is stationary, torques about ANY AXIS must sum to zero.
- Today, we'll talk about what happens if the torquees do not balance!

Why this is easy...

- Because all the laws of rotation are exact analogues of the laws of motion we've already done.
- All the equations are the same, as long as you

Swap angle θ for position x

Swap moment of inertia I for mass m

Swap angular velocity ω for velocity v

Swap torque τ for force

Let's talk about these new variables

- θ, ω and `I`

Circles...

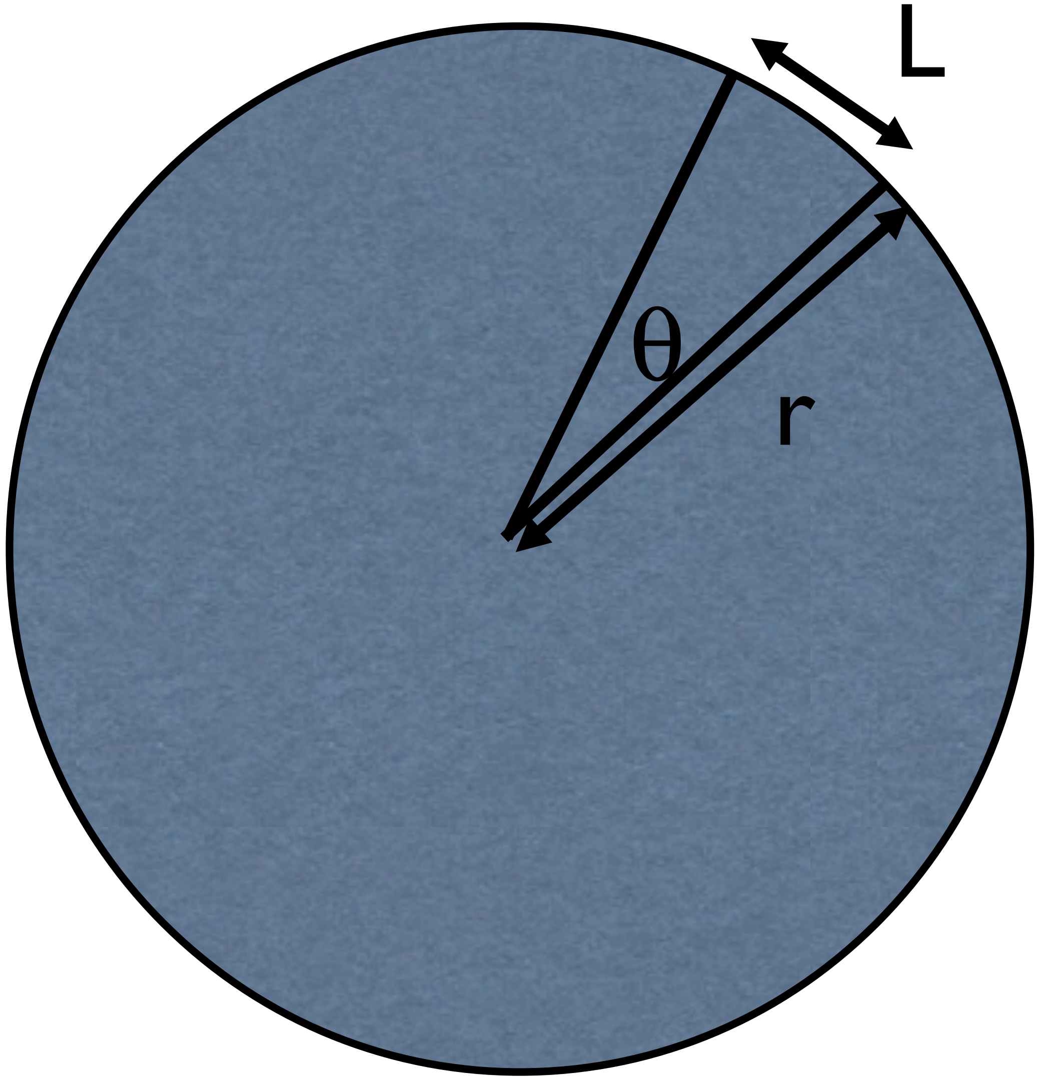
- Angles are measured around a point (the pivot or axis)
- If you are close to the axis, a given change in angle only means moving a small distance.
- Further away, a given change in angle means a large distance.
- Demonstration!

Angular velocity ω

- Is just the rate of change of angle θ. $\omega = \frac{d\theta}{dt}$
- θ is measured in radians, and ω is measured in radians per second.
- Remember, 2π radians is a complete circle, π/2 radians is 90°.
- To convert from degrees to radians, multiply by π/180.

If an object is going in a circle

- How far L does it go for a given change in angle θ (in radians)?



The best way to work this out uses ratios.

If you go all the way around the circle, $\theta = 2\pi$ and $L = 2\pi r$

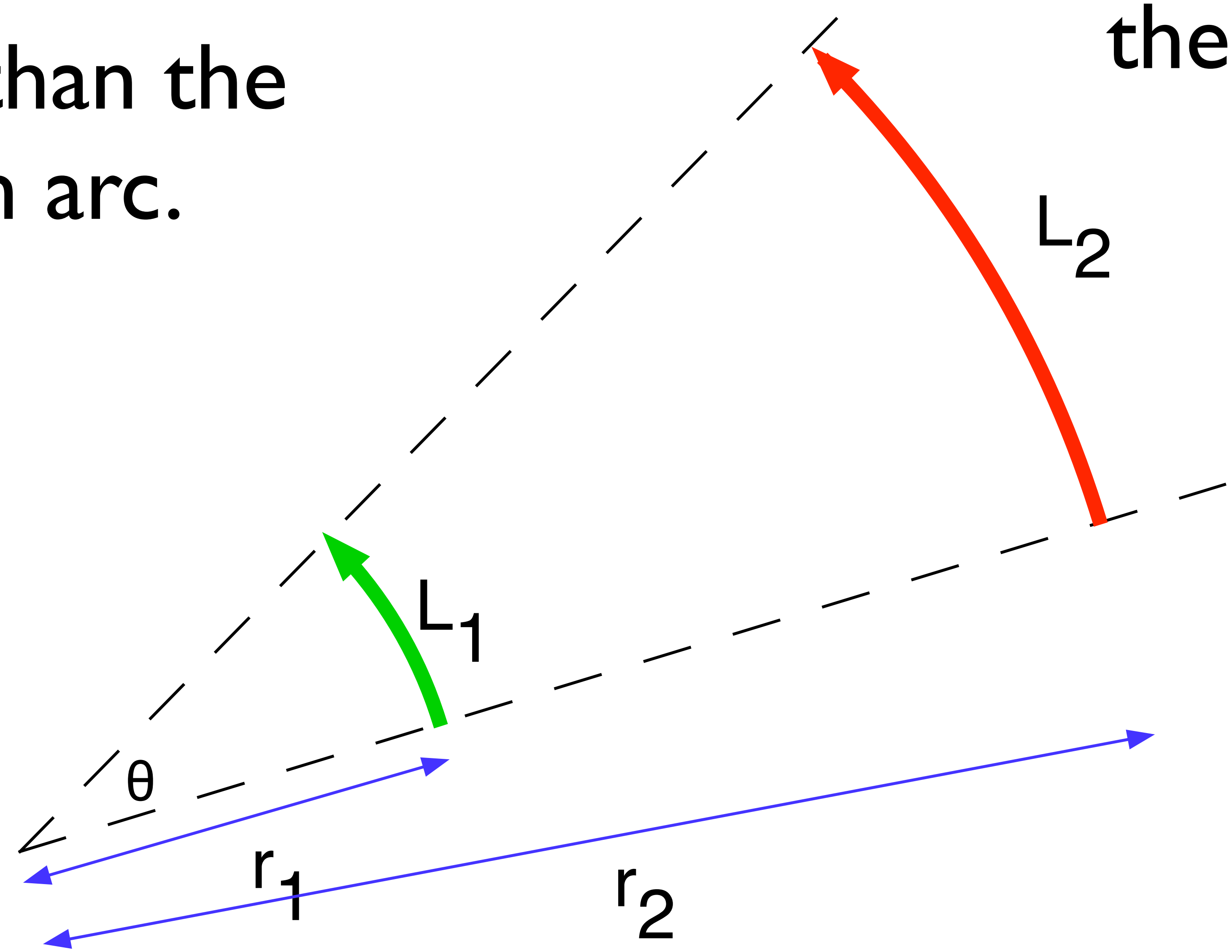
The ratio between θ and L should be constant (if you go twice the angle, you go twice the distance), so for all angles,

$$L = r\theta$$

Diagram at different distances

The red arc is longer than the green arc.

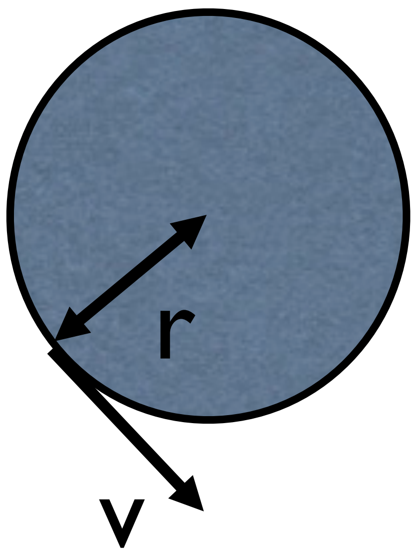
But both subtend the same angle at the axis



$L_1 = r_1\theta$

$L_2 = r_2\theta$

Relationship between linear and angular speed for wheels…



- If the wheel is rotating at angular speed ω, how fast (v) is a point on the circumference going?

1. $v = 2 \pi r \omega$
2. $v = 180 r \omega$
3. $v = r \omega$
4. $v = \pi r \omega$
5. $v = 2 \pi \omega / r$
6. $v = 180 \omega / r$
7. None of the above

The answer is $v = r\,\omega$

- In one full rotation, the point on the circumference must move a distance $2\pi r$
- And it rotates through an angle 2π radians
- So each radian of rotation moves the circumference by distance r.

Pro-tec

Two bikes question

- My bike has wheels with twice the diameter of my son's bike.
- If we are cycling down the road together, side by side…

1. The angular speed of my wheels is twice that of my son's.
2. The angular speed of my wheels is more than that of my son's, but not necessarily twice as much.
3. The angular speeds are the same.
4. The angular speed of my wheels is less than that of my son's, but not necessarily twice as slow.
5. The angular speed of my wheels is half that of my son's.

Half the angular speed

- The circumference of the wheels must be moving at the same speed.
- My radius is twice as large
- So the angular velocity must be half as big.

Bike speed

- I am cycling at 10 m/s on a bike with wheels of radius 20cm.
- What is the angular velocity of the wheels?

1. 2 radians/sec
2. 10 radians/sec
3. 20 radians/sec
4. 31.4 radians/sec
5. 62.8 radians/sec
6. None of the above
7. Not enough information provided.

The speed of the rims of the wheels (relative to the centre of the wheels)

…

- Must be the speed of the bike (or the wheels would be slipping on the road)
- $v = r\,\omega$
- $\omega = v/r = 10.0/0.2 = 50.0$ radians/sec

Moment of Inertia I

- This is how much something resists being rotated.
- What should it depend on?

Experiment

- Lengthen a walking stick by three times, and see how much longer it takes to rotate under the influence of the same force.

How much time?

1. Less than one third
2. One third
3. Between the same and one third
4. The same
5. Between the same and three times more
6. Three times longer
7. More than three times as long.

Answer

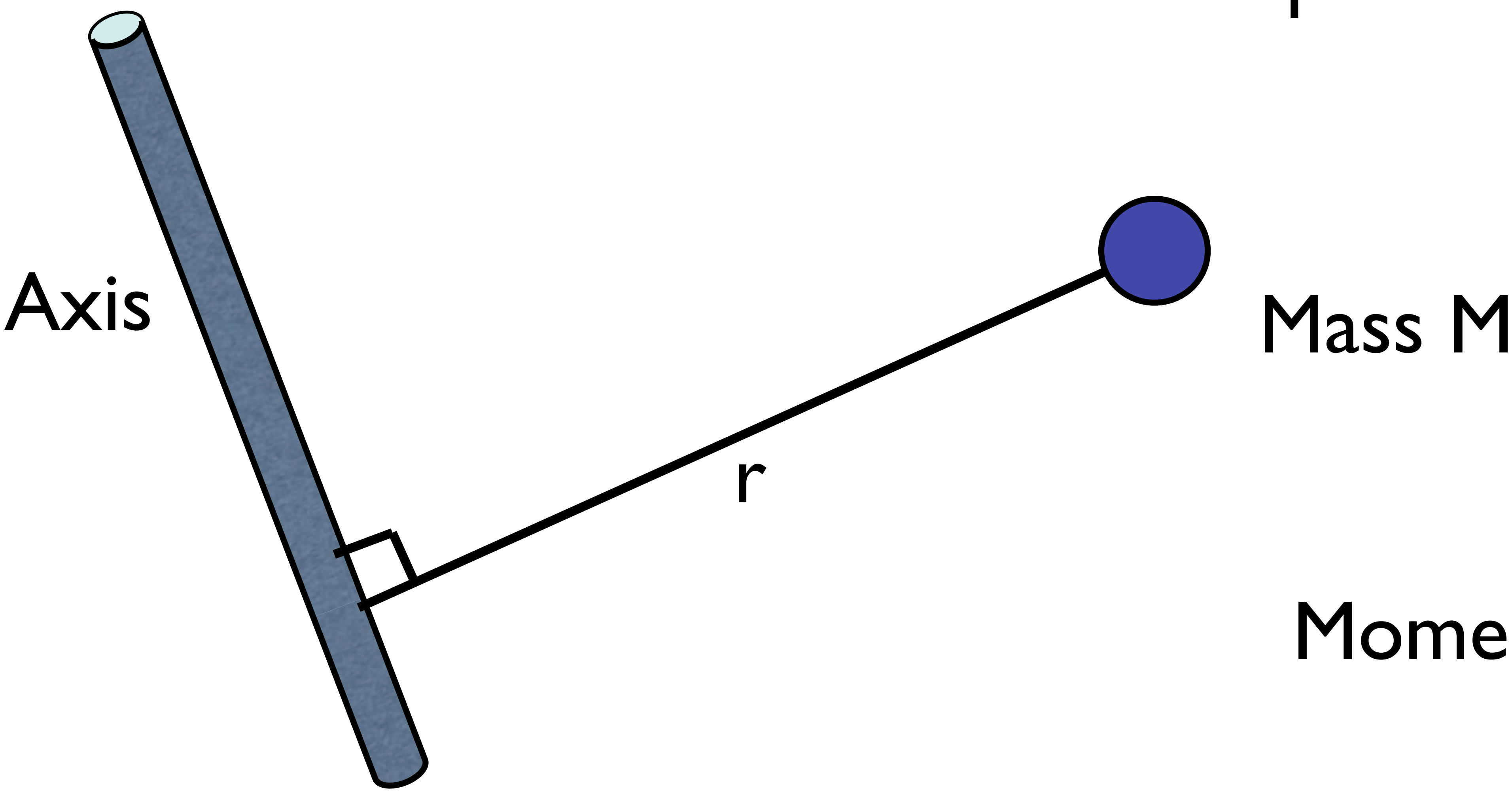
- About ten times longer!

Mass and Radius

- If something is heavier, it is harder to rotate.
- But most important of all is size. And not just any size - only size perpendicular to the rotation axis.
- How can we capture this?

Moment of Inertia

- If all the mass was in one place…



Moment of Inertia

$$I = M r^2$$

where r is the perpendicular distance between the nearest point on the axis and the mass

It depends on...

- The mass of something
- How far it is from the axis (very strongly!)

Moment of Inertia

- A 50g tennis-ball is on the end of a light-weight string of length 30cm.
- What is the moment of inertia of the ball if it is spun around in a circle on the end of this string?

1. 45,000 kg m^2
2. 1500 kg m^2
3. 0.15 kg m^2
4. 0.045 kg m^2
5. 0.015 kg m^2
6. 0.0045 kg m^2
7. None of the above
8. Not enough information provided

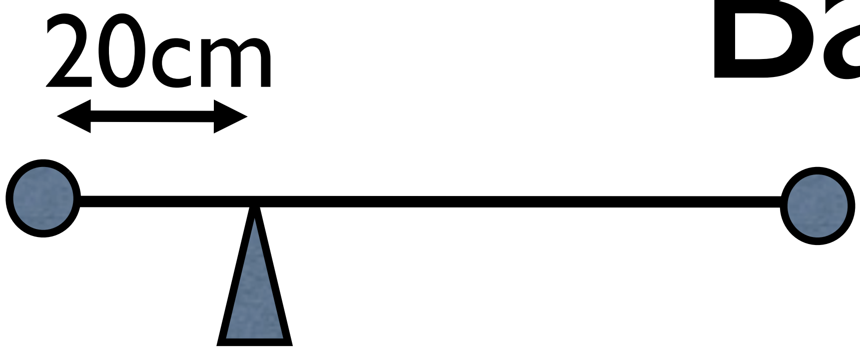
The answer…

- $I = M\ r^2 = 0.05 \times 0.3^2 = 0.0045$ kg m^2

Mass in several places

- You seldom have just one mass - what do you do if there are lots?
- Just add up the moments of inertia of all the different bits of mass.
- i.e. take each mass and multiply by the *square* of its perpendicular distance to the rotation axis.
- So small bits of mass a long way out have a BIG effect!

Baton Twirl



- A light-weight baton, 70cm in length has a mass of 100g on each end.
- If you are twirling it around a point 20cm from one end, what is its moment of inertia?
- You have one mass distance 0.5m from the axis, and one 0.2 m from the axis.
- Total moment of inertia is just the sum of these…
- … $0.1 \times 0.5^2 + 0.1 \times 0.2^2 = 0.029$ kg m^2

A continuous distribution of mass

- In reality, most objects are not a collection of tiny masses connected by light-weight rods - they have mass all over them.
- What do we do here?
- Approximate the true distribution of mass as lots of bits - work out the moment of inertial of each bit and add them up.

I’ll cover this later

- But for almost any situation, Google will find you the necessary equations.
- Look up the moment of inertia.

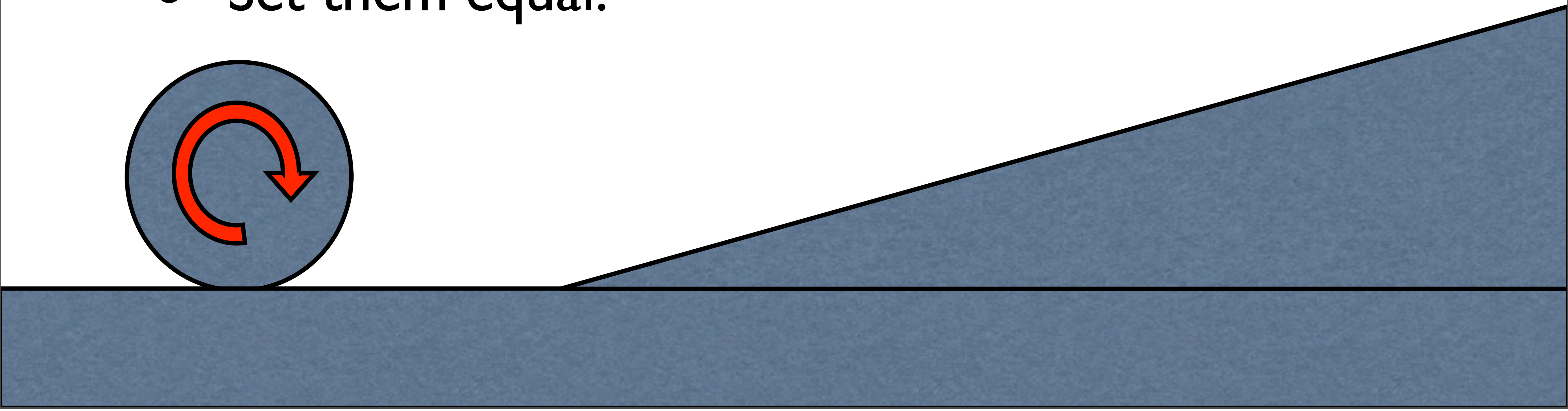
Two ways to solve rotation problems

1. Energy
2. Torque/Angular Momentum

- (Just like there are two ways to solve normal motion problems).

Energy example

- A ball is rolling along the ground, and reaches a hill. How high does it get?
- Initial energy is rotational (1/2 $I\omega^2$) and translational (1/2 mv^2).
- Final energy is potential (mgh)
- Set them equal.

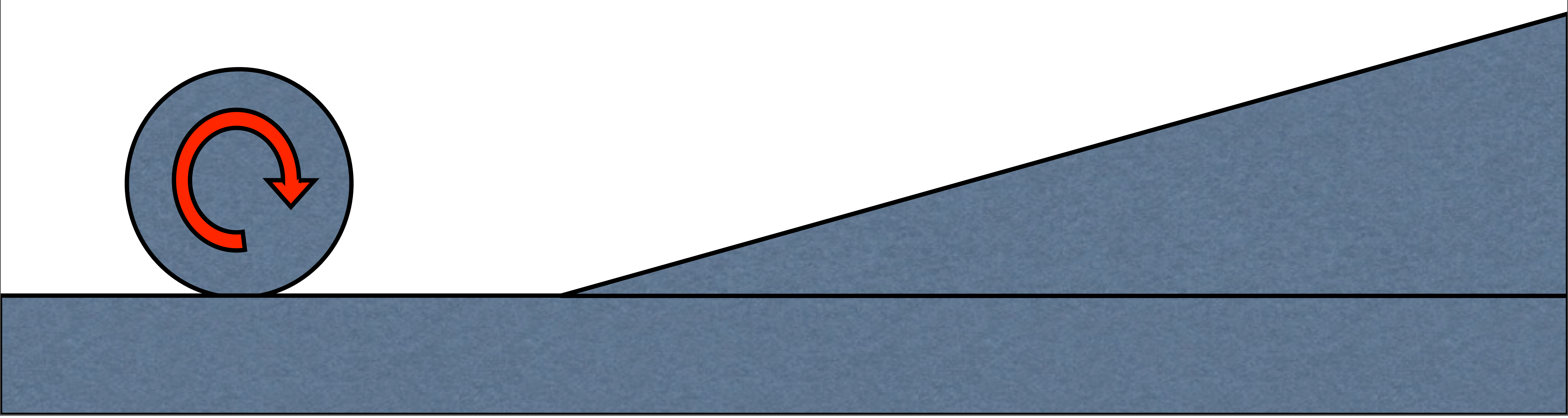


Note - two sorts of kinetic energy

- What do you do if an object is both rotating and moving?
- Use $1/2\ mv^2$ for the velocity of the centre of mass, and $1/2\ I\omega^2$ for the rotation about the centre of mass.

So -

- For a solid sphere (bowling ball, gold ball, cricket ball), Google tells us that $I=2mr^2/5$, or if the ball is a hollow sphere (soccer ball, tennis ball, ping-pong ball), $I=2mr^2/3$.
- Let's say it is a soccer ball.



Relationship between ω and v?

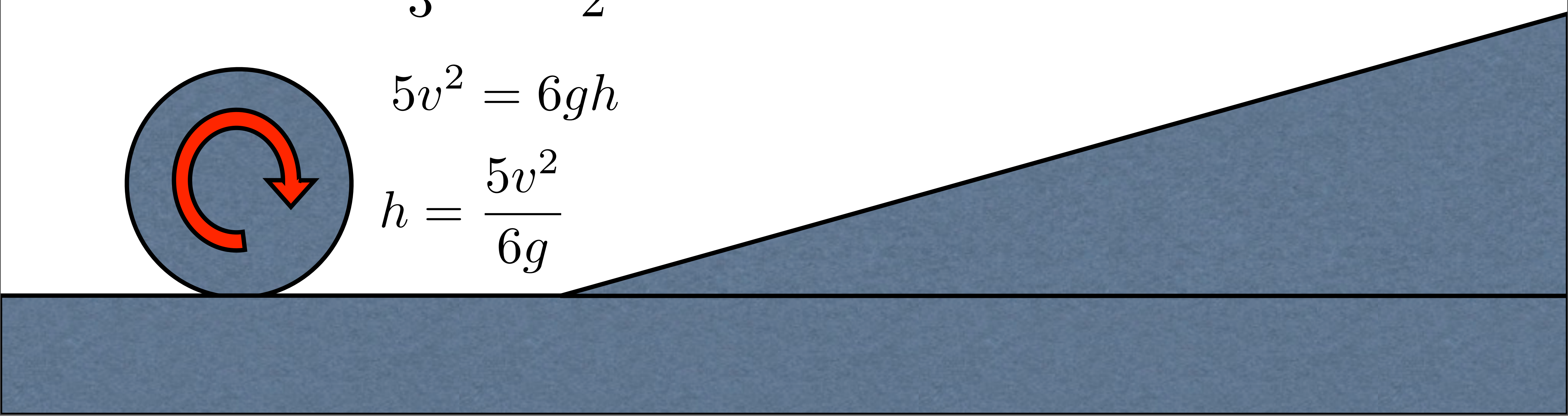
- If it is rolling and not sliding, r ω = v, as we discussed earlier.
- So our equation becomes…

$$\frac{1}{2}\frac{2mr^2}{3}\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$

$$\frac{mv^2}{3} + \frac{mv^2}{2} = mgh$$

$$5v^2 = 6gh$$

$$h = \frac{5v^2}{6g}$$



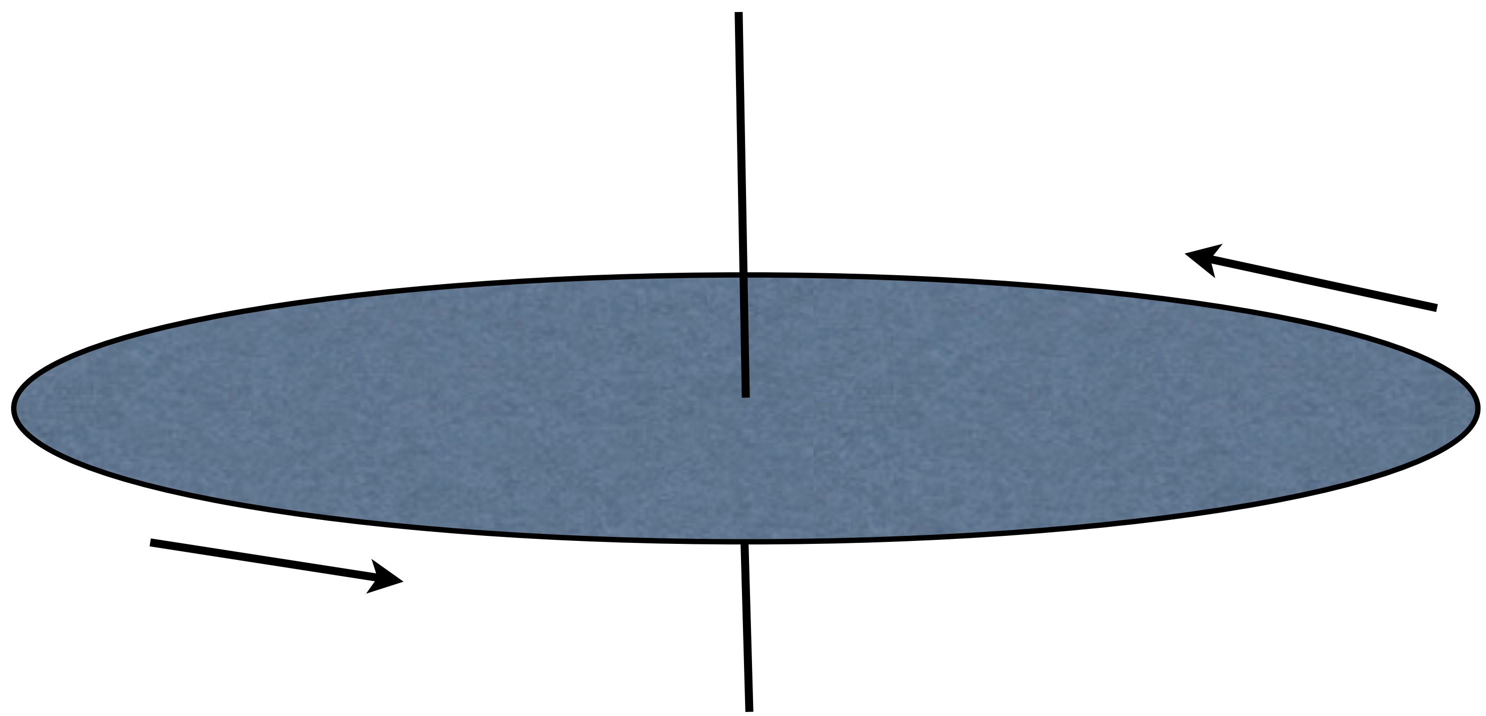
Torque/Angle Problem

- If you need to know exactly how long something takes, energy won't help you (damn!) and you will have to use the full torque/angle method.

Torque/Rotation Example

- A flywheel is being used to store energy in a factory. It is a solid uniform disk of mass M and radius R, and it spins at an angular velocity of ω.
- You have been asked to design a braking system which, in case of emergency, will bring it to a halt in time t.

Force?



Your brake applies a friction force F to the outside edge of the disk. How big must this force be?

You don’t need a net torque…

- To keep something spinning…
- (though if there is friction you will need to apply a torque to balance out the friction torque)
- You need to apply a torque to **change** the angular speed of something - to speed it up or slow it down.

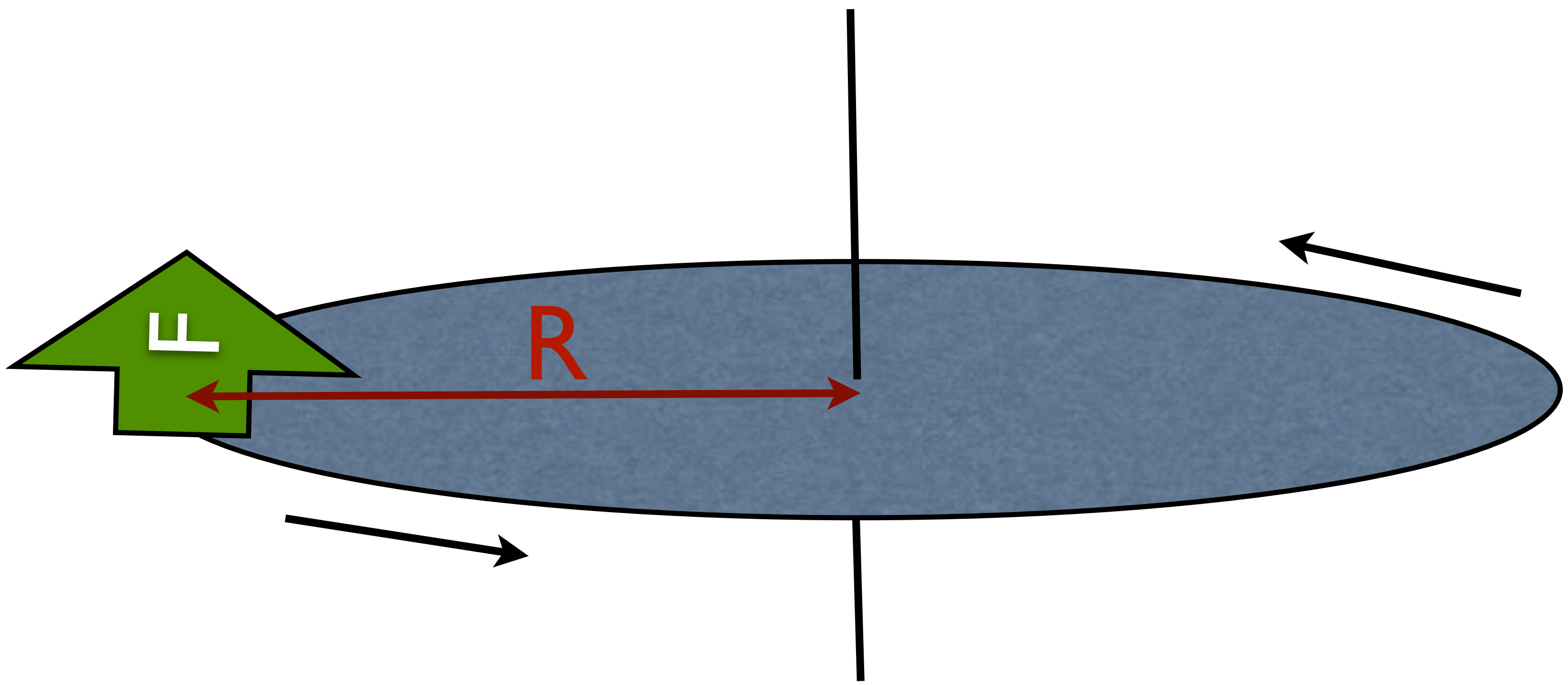
Rotation of Earth

There is no torque on the Earth.
It's been spinning for four billion years due only to the initial spin it got when if formed.

Well, actually, there is a torque

- From the moon, via tides.
- It is slowing down the Earth's rotation and making the day longer.
- The day probably started off as only a few hours long.

Torque?



Torque τ=FR

Analogy

- If we knew the force and wanted to know how long something would take to stop, we would use F=ma to work out the acceleration.
- Then use the acceleration to work out how long until the speed was zero.

Here...

Use $\tau = I\frac{d\omega}{dt}$ which is the angular equivalent of

F = ma, to work out $\frac{d\omega}{dt}$ which is the rate of change

of the angular speed ω. Divide ω by $\frac{d\omega}{dt}$ to find the

time t needed to stop.

Clicker Question

- If the angular speed is 10 radians per second, and the rate of change of angular speed is -2 radians per second, how long until the object stops spinning?

Answer

- 5 seconds... (10/2)

Moment of Inertia of a disk

http://en.wikipedia.org/wiki/List_of_moments_of_inertia

- tells us that for a uniform disk, $I = \frac{1}{2}mr^2$

SO...

$$\frac{d\omega}{dt} = \frac{\tau}{I} = \frac{FR}{\frac{1}{2}mR^2} = \frac{2F}{mR}$$

And...

time to stop t is $t = \frac{\omega}{\frac{d\omega}{dt}} = \frac{\omega m R}{2F}$

Rearrange to make F the subject

$$F = \frac{\omega m R}{2t}$$

Plausibility check - a faster initial rotation, a wider disk, a more massive disk or less time to stop all necessitate a bigger braking force.

Summary

- If you apply a torque to an object, its rotation rate will change.
- You can solve problems like this using energy or torque and angular acceleration.
- The latter is an exact analogy of force/ momentum problems.