Motion in an Circle and Oscillation

Two Special Cases

Course news

- Labs start tomorrow
- Clickers will now be used for assessment. You need to have a "U" in front of your student number in the clicker. If you can't join the class, come see me now!
- Class reps - introduce yourselves.

Course Reps Nominated

- Samantha Cheah
- Raj Srilakshmi
- Ellen Rykers
- Lachlan McGinness
- Sarah Biddle

Momentum and Force

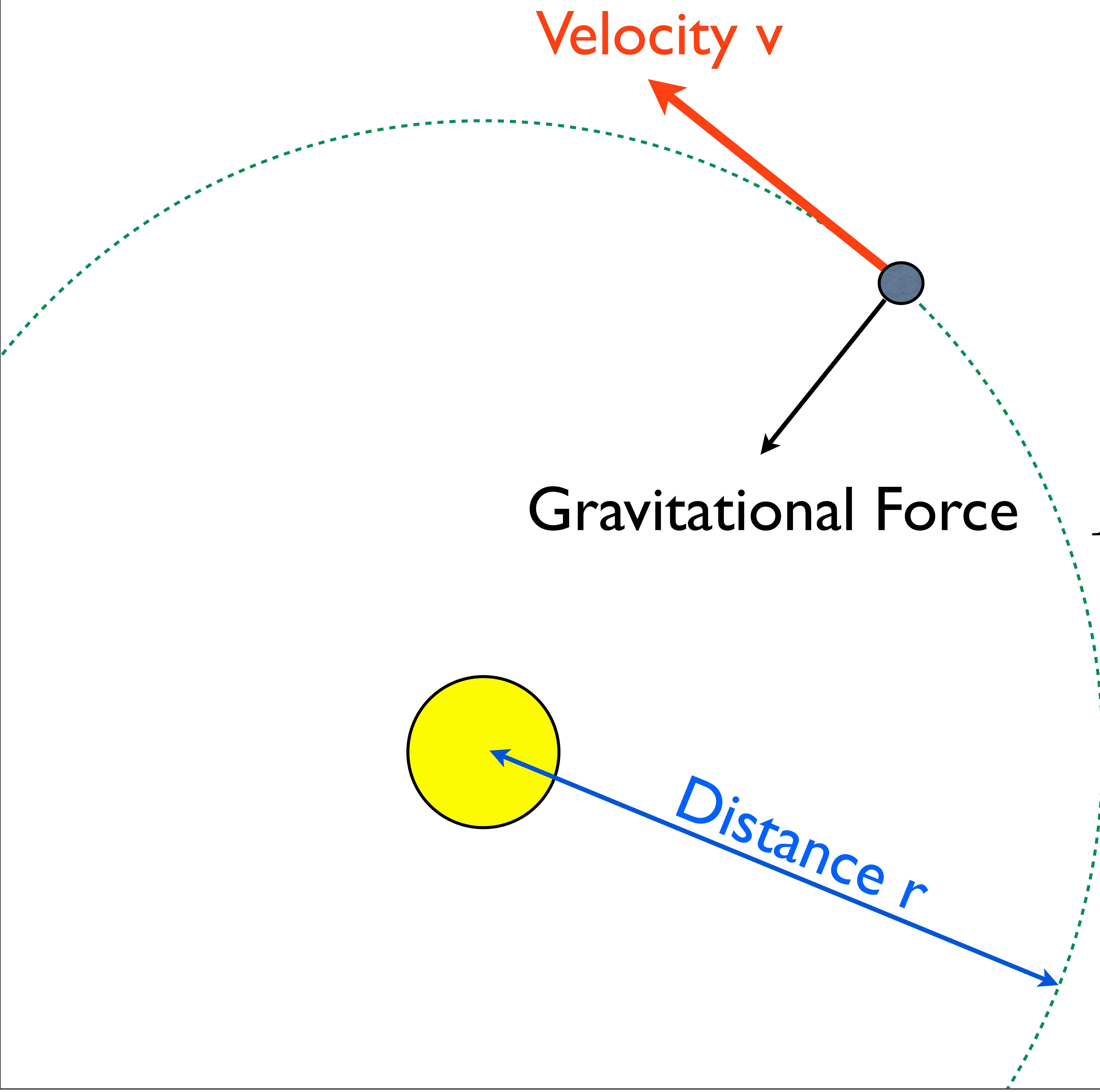
- We will talk about two special cases - circular motion and oscillation.
- Then we will start dealing with the general case.

Circular motion

- Remember - if a force is applied that is always sideways, an object will move in a circle.

Example - Orbits

- If one object (say the Space Station) is in a circular orbit around another, much larger object (say the Earth), the larger object's gravity must be supplying the necessary (centripetal) force to keep the space station moving in a circle.



$$F = \frac{GMm}{r^2}$$

To stay in a circular orbit, this gravitational force must supply the necessary centripetal force $F = \frac{mv^2}{r}$

so...
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Cancel masses and one of the r's

$$\frac{GM}{r} = v^2$$

Rearrange to find v

$$v = \sqrt{\frac{GM}{r}}$$

Centrifugal Force

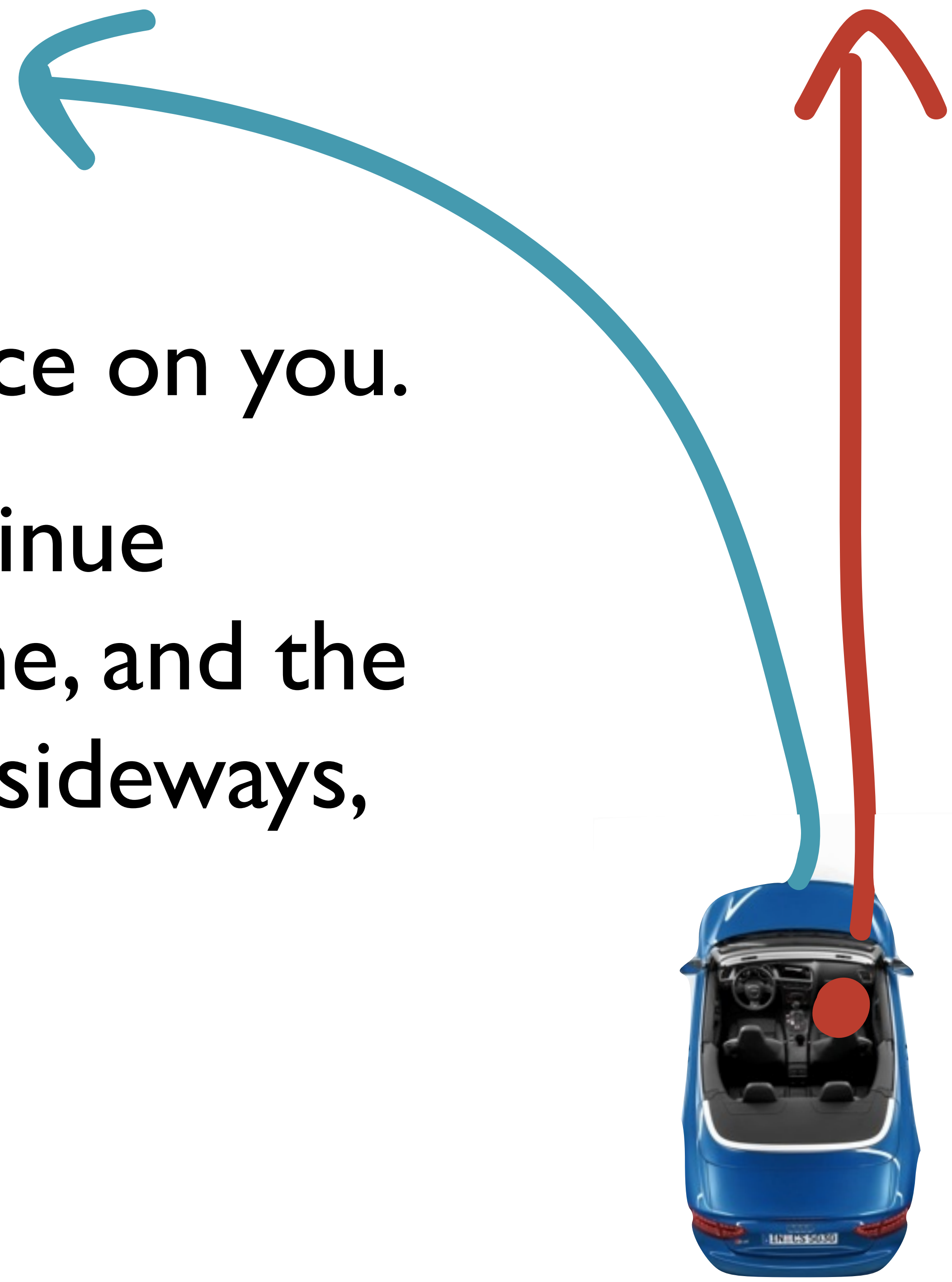
- A particularly confusing topic

Clicker Question

You are a passenger in a car and not wearing your seatbelt. Without increasing or decreasing its speed, the car makes a sharp right turn, and you find yourself colliding with the left-hand door. Which is the correct analysis of the situation?

1. Before and after the collision, there is a leftward force pushing you into the door
2. Starting at the time of the collision, the door exerts a leftward force on you
3. Both of the above
4. Neither of the above

The Answer



- The door exerts a force on you.
- You are trying to continue moving in a straight line, and the door pushes into you sideways, forcing you to turn.

Centrifugal Force

- Centrifugal force is even more imaginary than centripetal force.
- There is no outward force when you go around a circle.
- You are just trying to continue in a straight line and being prevented from doing so by some force (which might be due to gravity or friction or the door, acts towards the centre and has magnitude $\dfrac{mv^2}{r}$

SImilarly for “g”-forces

- When you speed up or slow down there is no “g”-force. You are being pushed by your chair or the dashboard.
- This push is what is changing your speed.

Crucial Facts

- Special case - a force that is constant in magnitude but perpendicular to the motion.
- Result - motion in a circle.
- The force points at the centre of the circle.

$$F = \frac{mv^2}{r}$$

Spring force

- This is another special case - a situation you almost never meet in the real world, but which can be solved without the need for a computer.

Spring Forces

SImple Harmonic Motion

“Ideal Spring”

- It has a “Natural” or “unextended” length.
- Whenever you pull it away from this length by a distance D, it exerts an opposing force $F = -kD$

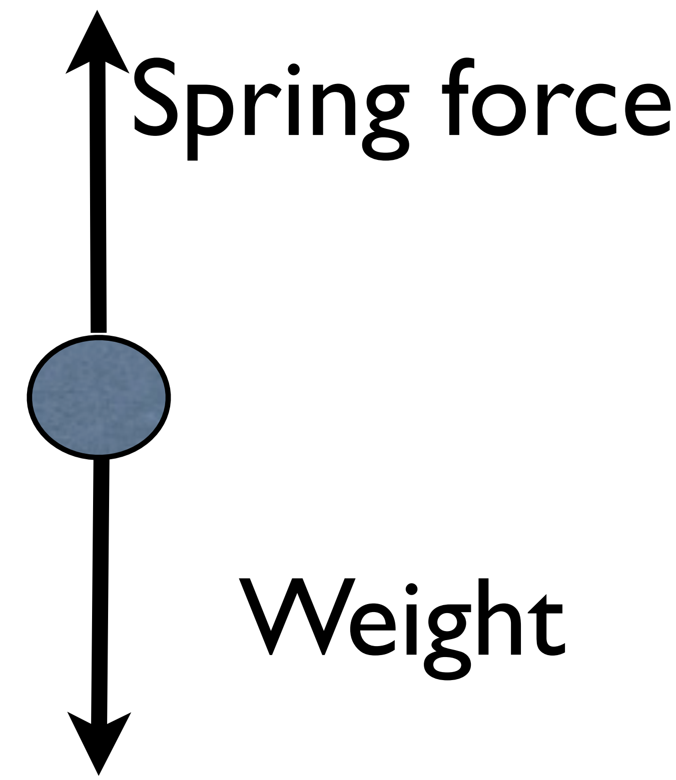
where k is the “Spring Constant”

Vertical weight calculation

- Example - a 50 g weight is hung from a spring of constant k=3.0 N/m.
- By how much does it stretch?

Draw a free-body diagram for the weight

- This is a diagram just showing the weight, as a dot, and the forces ACTING ON IT



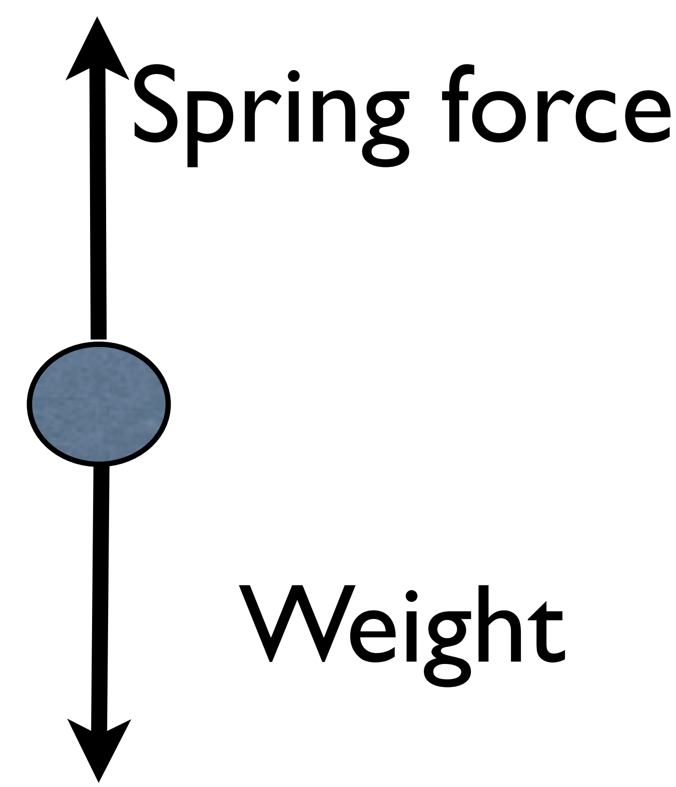
As it's hanging still...

- Forces must balance. So the weight and the spring force must be the same

$$mg = kD$$

Rearrange to get D

$$D = \frac{mg}{k}$$



So this gives how much the spring stretched.

Why are we worried about this?

- Because while ideal springs are rare, forces which always pull towards a point are common.
- Such as chemical bonds
- Any elastic behaviour
- So it's worth getting used to this sort of force.

Motion attached to a spring

- We’ve seen how to calculate a static situation with a spring.
- But what if something is moving while attached to a spring?

Vertical spring-mass system

- VPython simulation, spring_vertical.py

Oscillation

- The net force is towards the equilibrium position.
- It accelerates towards it.
- But thanks to momentum, it overshoots. The force is now backwards and slows it to a halt.

Energy

- A constant interplay between kinetic and spring energy (with a little gravitational potential energy thrown in for good measure)

Very general behaviour

- Whenever you get any sort of force which tends to push things back into place.
- Usually need a computer to solve exact motion, but if you assume the spring is ideal (seldom the case in reality) you can solve it.

Analytic Solution

- I'll show you the mathematical solution in this idealised case.
- But first - what would you expect to determine how rapidly it oscillates?
- What makes it oscillate faster?

Clicker Question

- What makes it oscillate faster?
- The spring constant?
- The mass?

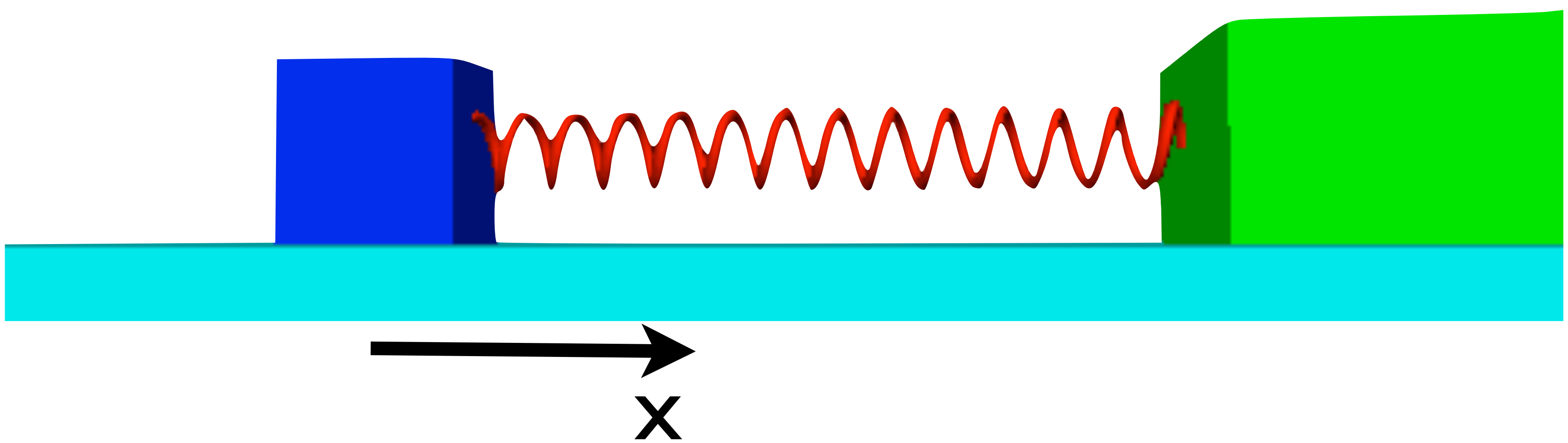
Answer

- A stiffer spring - pushes back harder
- A lighter mass - accelerates faster.

Horizontal

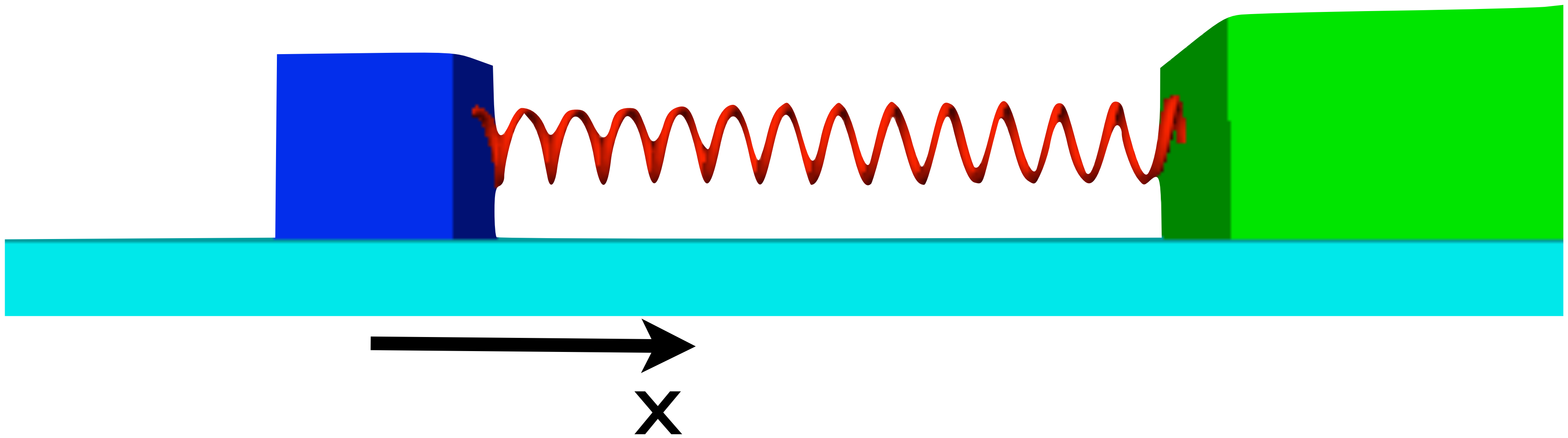
- To make the maths simpler, let's take a horizontal spring-mass system, with the mass sliding along a frictionless surface.
- (the result is the same as for a vertical system but the argument is a bit simpler)

Coordinates



- Let's call the position of the weight x, and measure it from the spring's rest position.
- (Once again you can use any axes you like and will get the same result, but it makes the calculation messier).

Force



- The only horizontal force acting is the spring force $F = -kx$

As we know the force, we can work out the acceleration using F=ma

$$a = -\frac{k}{m}x$$

Calculus

- So we know the acceleration. But what about the velocity v or position x?
- Luckily, we know that acceleration is defined as the rate of change of velocity.
- So

$$a = \frac{dv}{dt} = -\frac{k}{m}x$$

Position

- And velocity is defined as the rate of change of position, so $v = \frac{dx}{dt}$

This means that acceleration a is

$$a = \frac{dv}{dt} = \frac{d\left(\frac{dx}{dt}\right)}{dt} = \frac{d^2x}{dt^2}$$

So acceleration is what you get when you differentiate position twice with respect to time.

So we now know that...

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

- k and m are just constants. So this is telling us that if you differentiate x twice, you get x back again, albeit multiplied by a constant.
- Can you think of any functions that when you differentiate themselves twice are unchanged (apart from a constant?)

What appears in its own second differential?

- How about Cosine?
- Let's try x = A cos(ωt), where A and ω (omega) are constants, currently unknown.
- Let's try differentiating this twice

$$x = A\cos(\omega t)$$

$$\frac{dx}{dt} = -A\omega\sin(\omega t)$$

$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t) = -\omega^2 x$$

It works!

- Compare $$\frac{d^2x}{dt^2} = -\omega^2 x$$

with the spring acceleration equation we got earlier

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

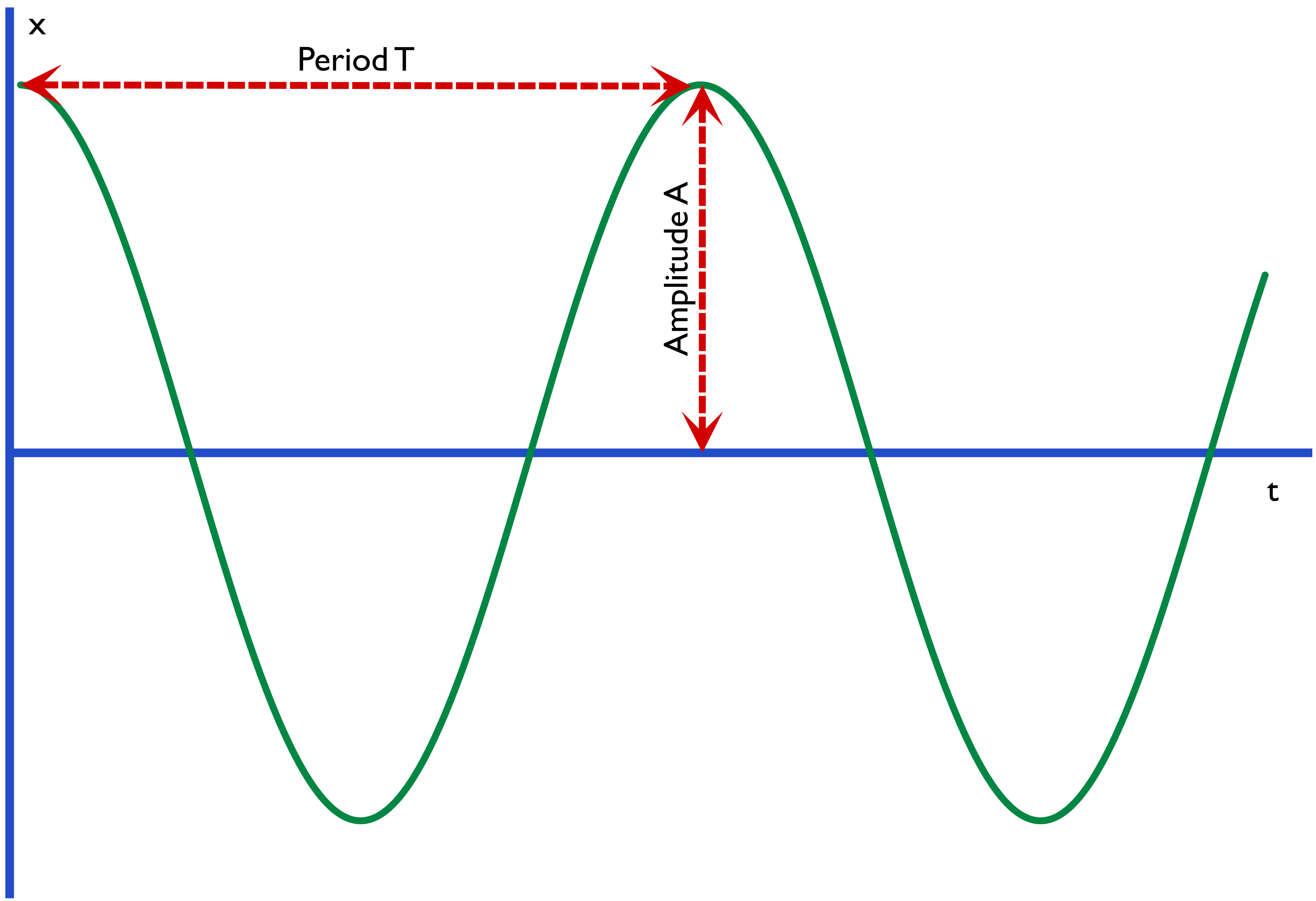
Identical, as long as we make $$\omega = \sqrt{\frac{k}{m}}$$

So the answer is...

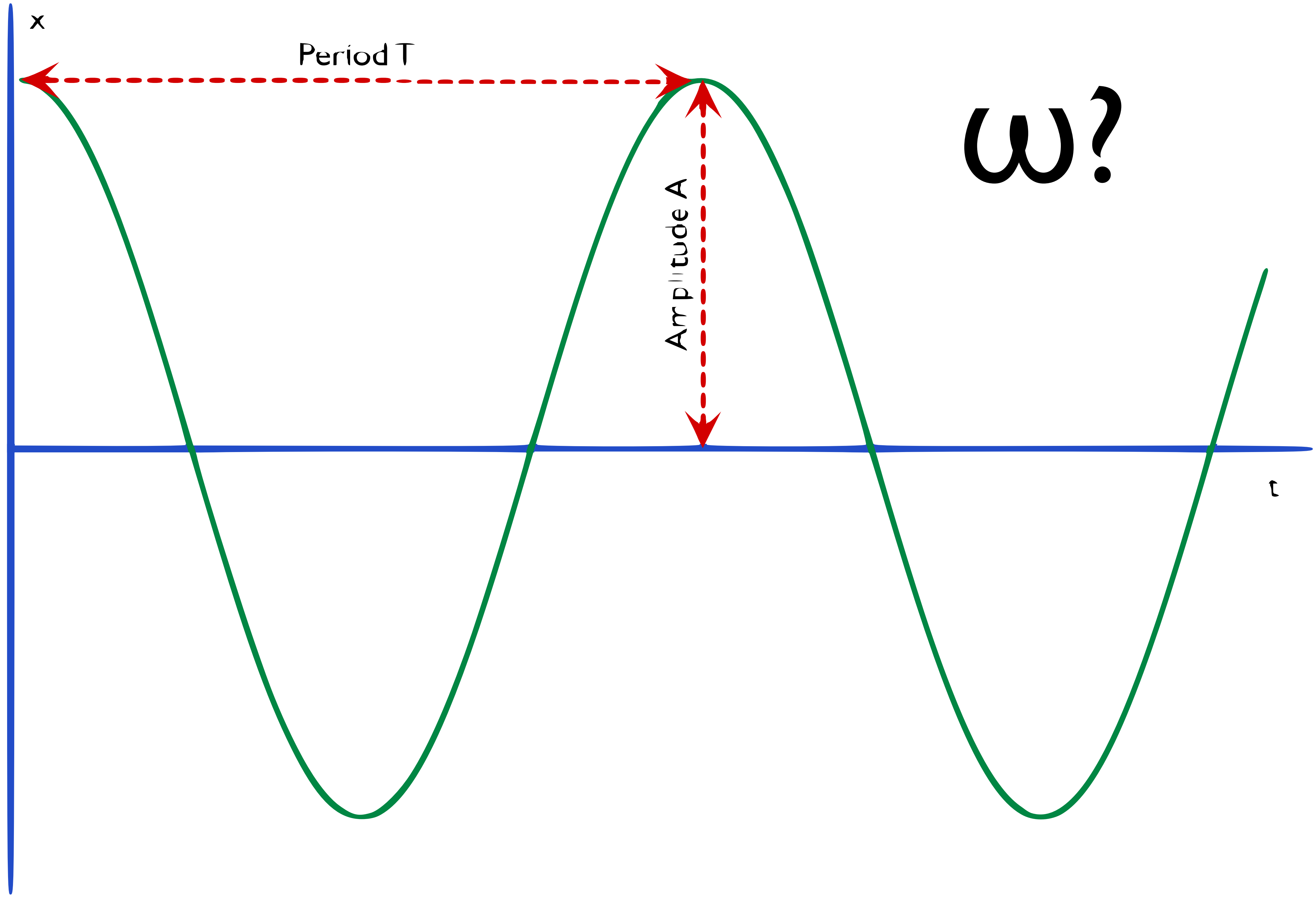
$$x = A\cos(\omega t) \quad \text{where} \quad \omega = \sqrt{\frac{k}{m}}$$

- This whole derivation should remind you of the projectile motion one.
- Write down F=ma, and integrate twice to get position versus time.
- This is called "simple harmonic motion"

What are ω and A?



A is the amplitude of the oscillation - how far it goes ON ONE SIDE of the equilibrium position



ω is the angular frequency, and is measured in radians per second. As 2π radians is a complete circle, this corresponds to the period T above.

Period and Frequency

So the angular frequency $$\omega = \sqrt{\frac{k}{m}}$$

- The period T (time to repeat) is $$T = \frac{2\pi}{\omega}$$

- The frequency (in cycles per second, also known as Hertz, Hz) is $$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Very Useful

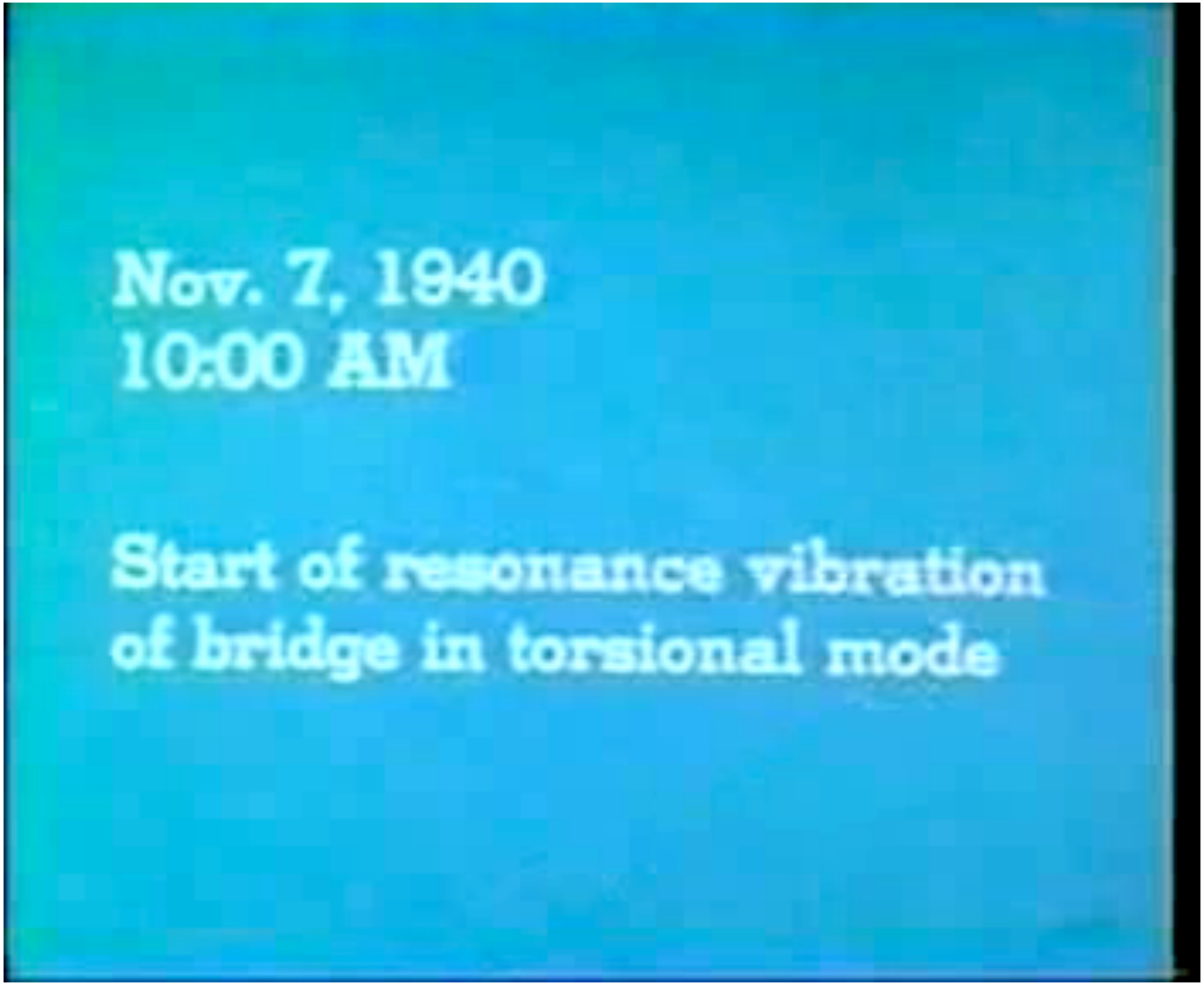
- You will come back to these quantities time and time again, as they are fundamental in waves, interference and all sorts of vibrations.

Resonance

- One final feature.
- An oscillating system like this is peculiarly responsive to outside wiggles at its natural frequency.
- This is called resonance.

Vpython simulation

Tacoma Narrows



SkyMapper

- Currently being commissioned
- Has a resonance problem. The cryocoolers are resonating with the secondary mirror (we think)

Carbon Dioxide

- VPython simulation

Key Points

- Whenever you get a force that pushes back towards some equilibrium position, you probably get vibrations.
- You can work out the frequency of oscillations if you know how strong the restoring force and how big the inertia of whatever is being vibrated.