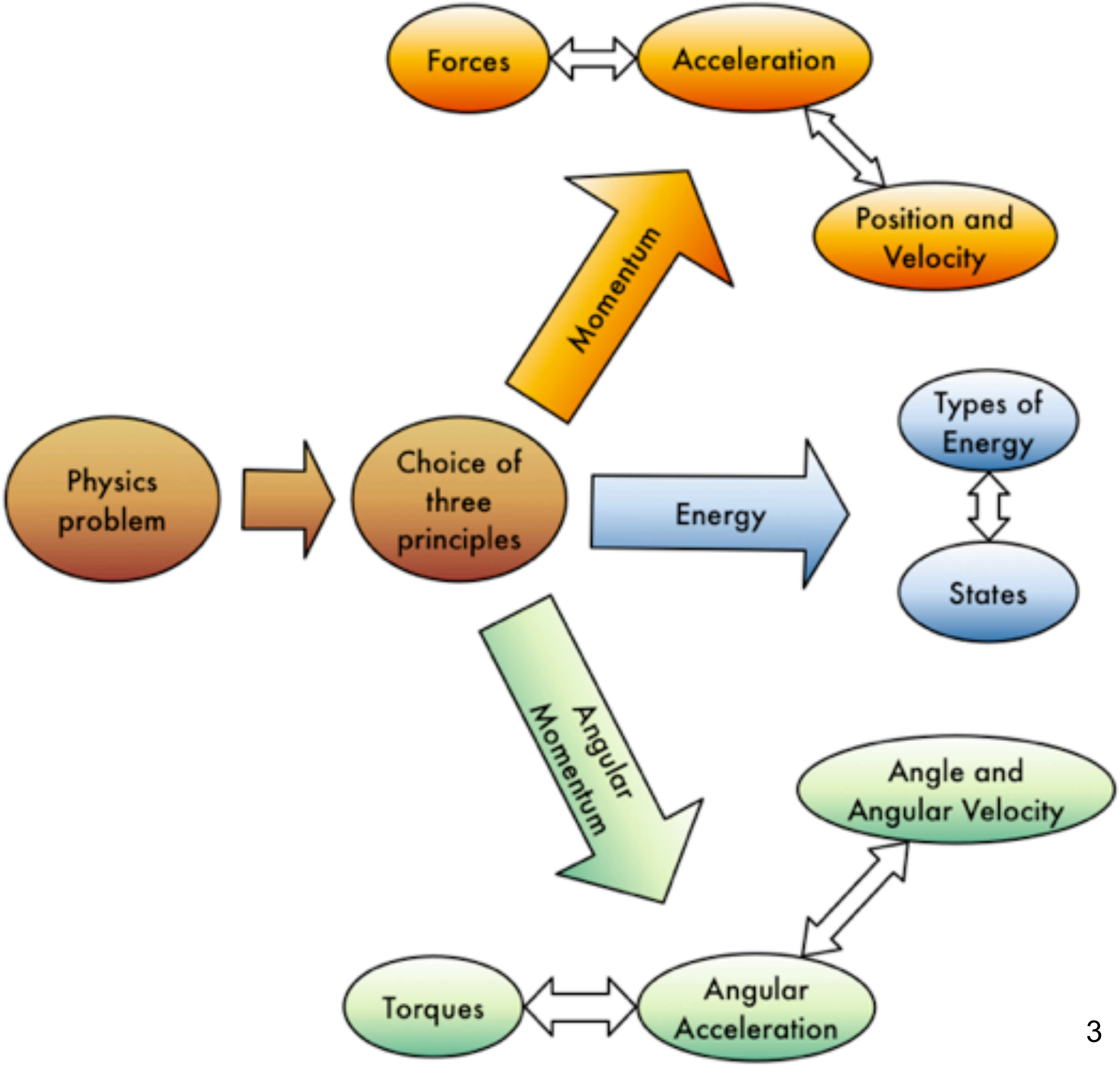
# Momentum

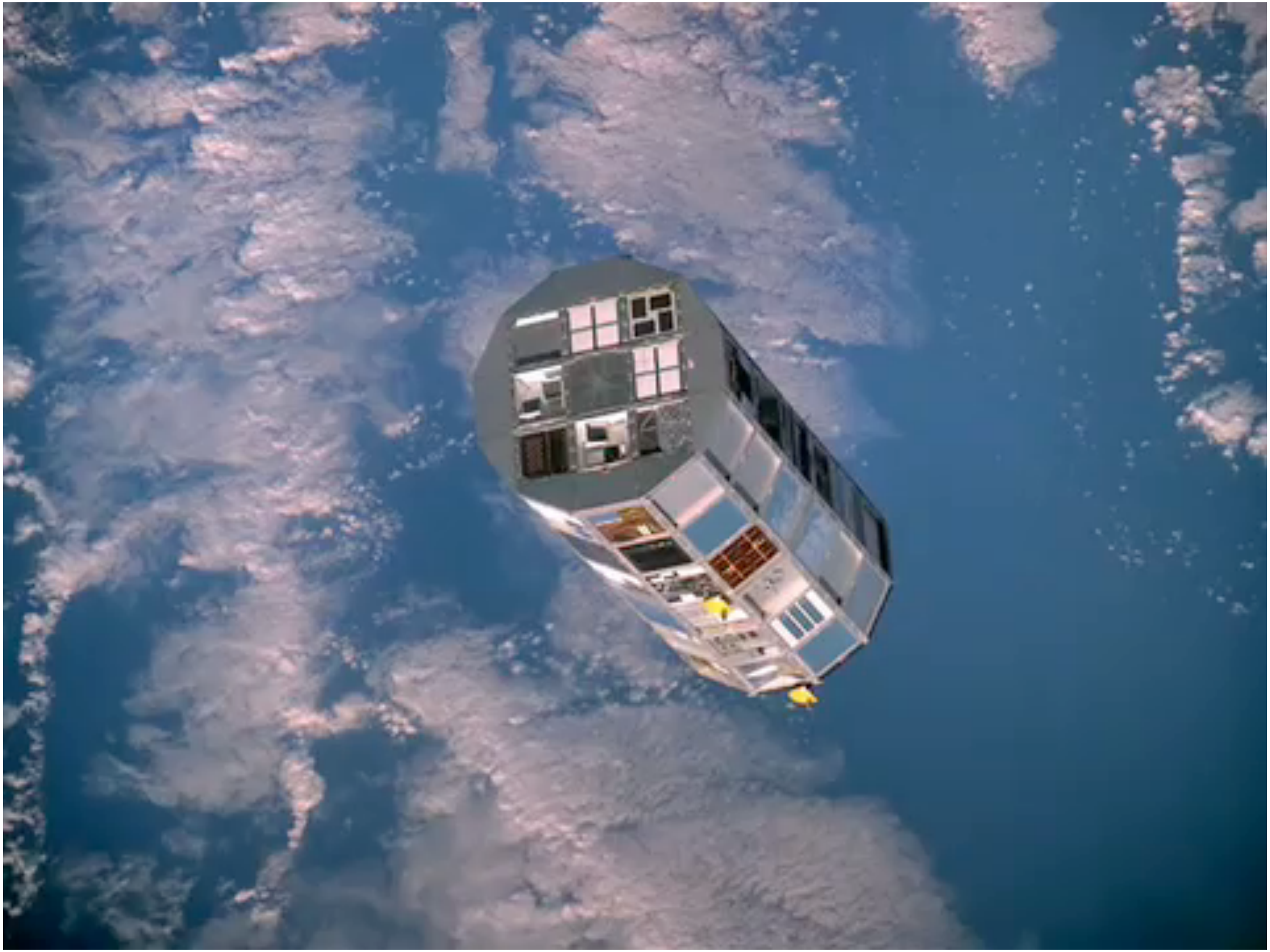
# Quick Introduction to Momentum Principle

- We will come back to all of this - this is just a taster.
- The momentum principle is another way of saying ‘Newton’s Laws”
- It is one of the three great principles of mechanics (along with energy and angular momentum)


Forces
Acceleration
Momentum
Position and Velocity
Physics problem
Choice of three principles
Energy
Types of Energy
States
Angular Momentum
Angle and Angular Velocity
Torques
Angular Acceleration

# Momentum

- Fundamental idea - things keep moving in a straight line unless you push them.



# Basic principle not at all obvious on Earth

- On Earth, things tend to stop unless you keep pushing them...
- Had we evolved in space, it would have been much clearer that they keep going.

# Momentum

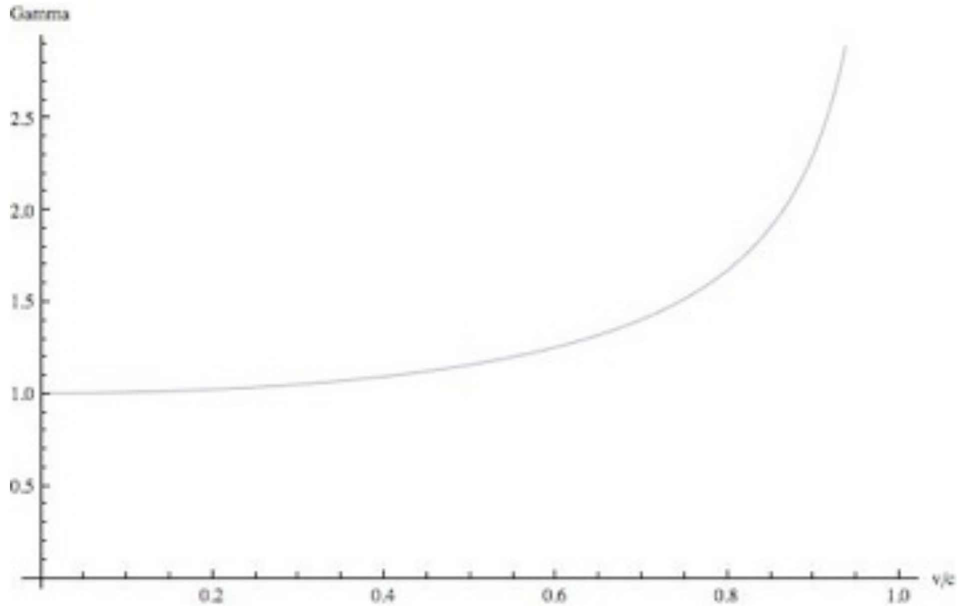
- Momentum is given by

$$\vec{p} = \gamma m \vec{v} \qquad \text{where} \qquad \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

- It's a vector - the direction matters!
- The faster you travel, or the heavier you are, the more your momentum.

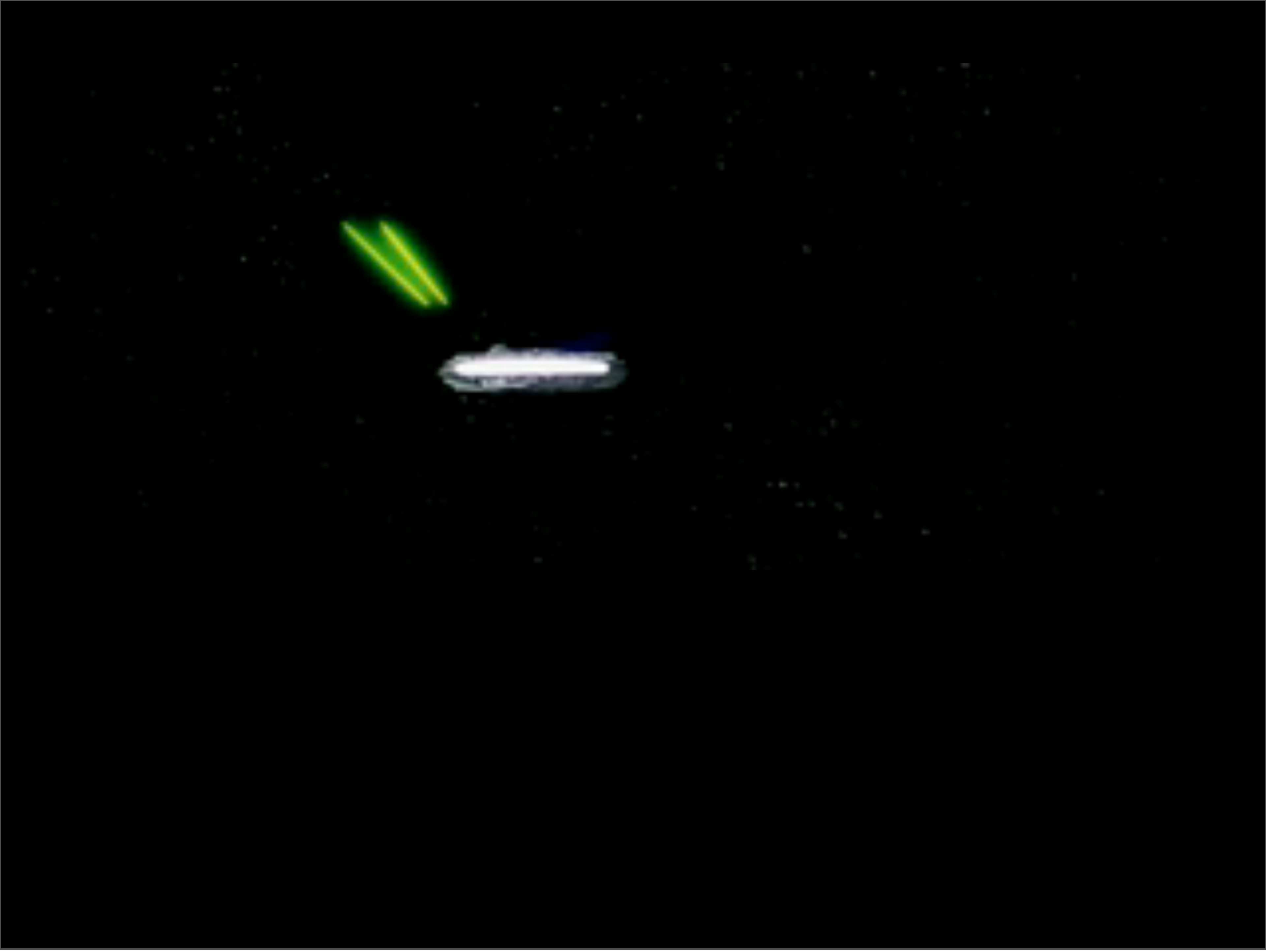
# What is this γ (Gamma)?

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$




You are probably familiar with momentum - mass times velocity - but the real equation has this extra term γ in it. This comes from Relativity - for speeds much less than light it is very very close to one and can be ignored.

As you get close to the speed of light it tends to Infinity - which is why you can't go faster than light.



# Don't normally need to worry about Gamma

- Unless you are dealing with remarkably fast motion!

- So for most purposes, p=mv

# Changing Momentum

- Momentum only changes if you apply a force
- You can use the following fundamental equation - that the rate of change of momentum is equal to the force.

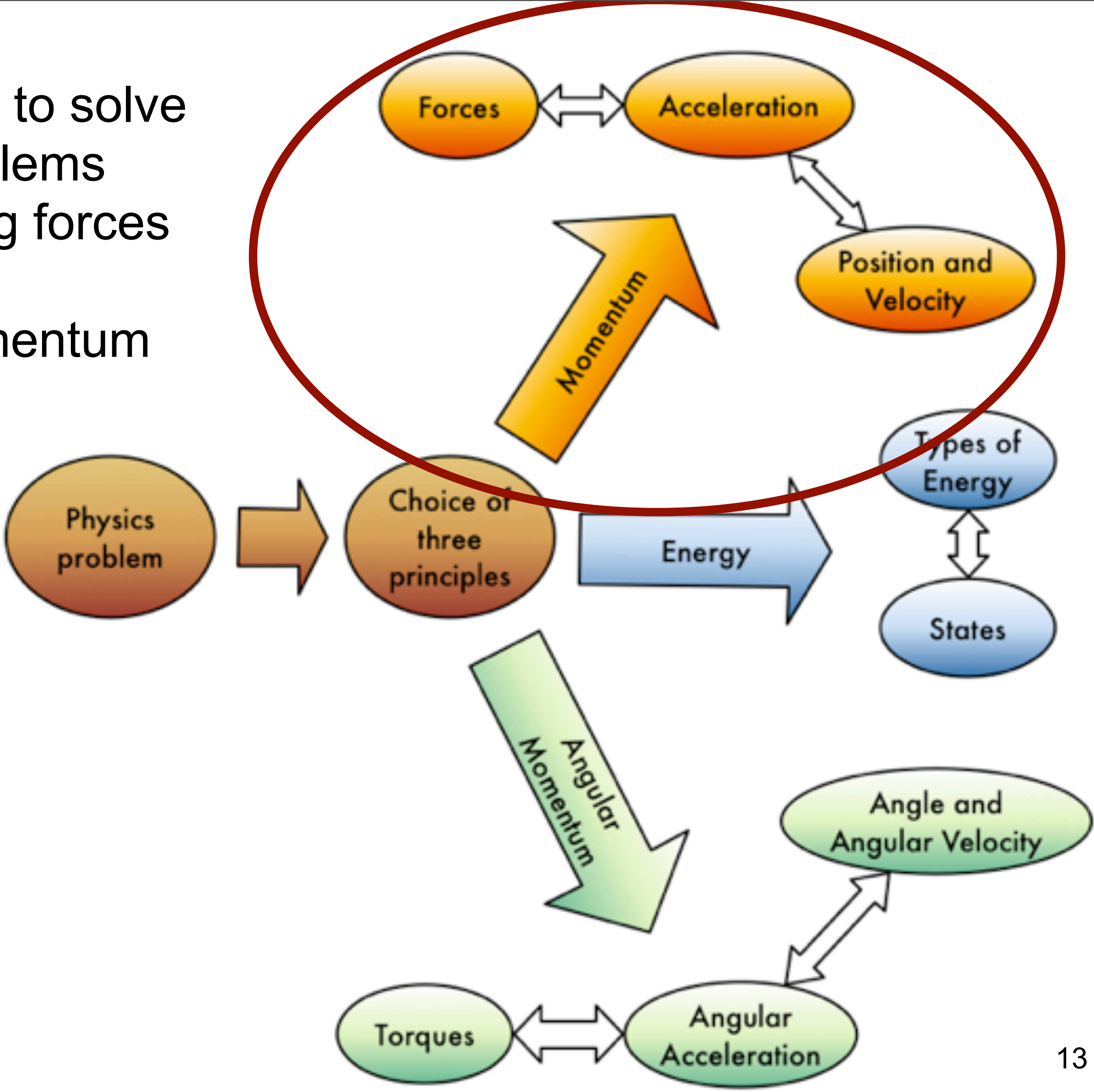
$$\frac{d\vec{p}}{dt} = \vec{F}$$

# Substitute in p = mv

$$\frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma = f$$

- So you see that (for speeds much below the speed of light), the rate of change of momentum is mass times acceleration, so this equation boils down to the familiar F=ma

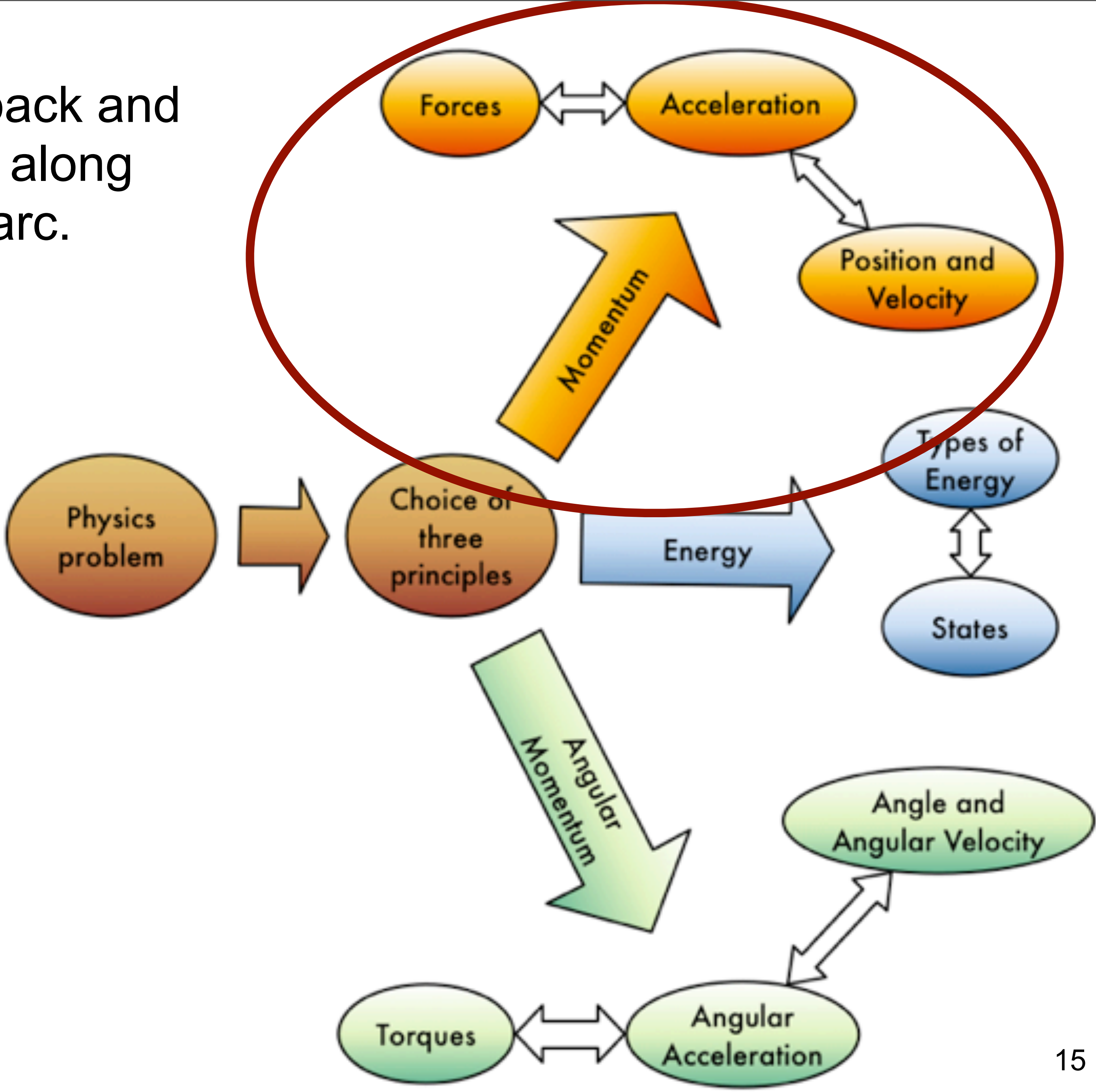
# How to solve problems using forces and momentum

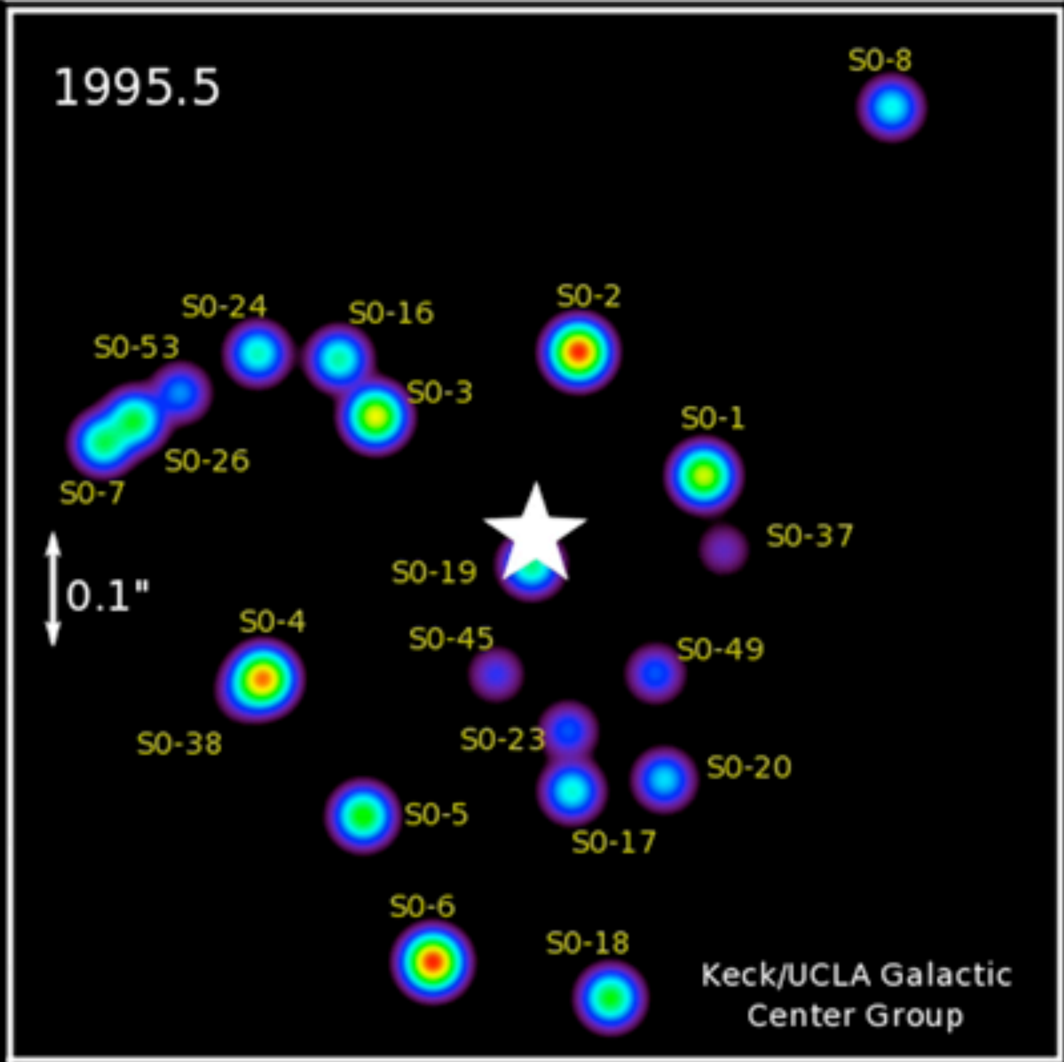


# Two methods

- Sometimes you will know the forces. Use F=ma to work out the acceleration, integrate the acceleration to find the velocity and position.
- Sometimes you know the velocity or position. Differentiate to work out the acceleration. Use F=ma to work out the forces.

Go back and forth along this arc.





1995.5
S0-8
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S0-23
S0-20
S0-5
S0-17
S0-6
S0-18
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# Why it’s hard

- Forces and accelerations are usually variable.
- They are vectors and can point in many directions.

- In general, the only way to solve problems like this is step-by-step using iteration on a computer - we’ll do that later.

# Special Cases

- However there are some special cases which can be solved using maths.

1. No motion - statics.
2. Constant force (both in direction and magnitude) - projectile motion
3. Force of constant magnitude at right-angles to motion - centripetal force.
4. Ideal spring force - simple harmonic motion.

# In the real world...

- You seldom meet any of these cases (except perhaps statics).
- Until about 50 years ago, however, we didn’t have computers so these were the only situations we could solve.
- Textbooks are full of these special cases.

# Today - projectile motion

- We will come back to the other special cases, and iteration, later in the course.
- Projectile motion is the situation in which forces on an object are constant in both magnitude and direction.

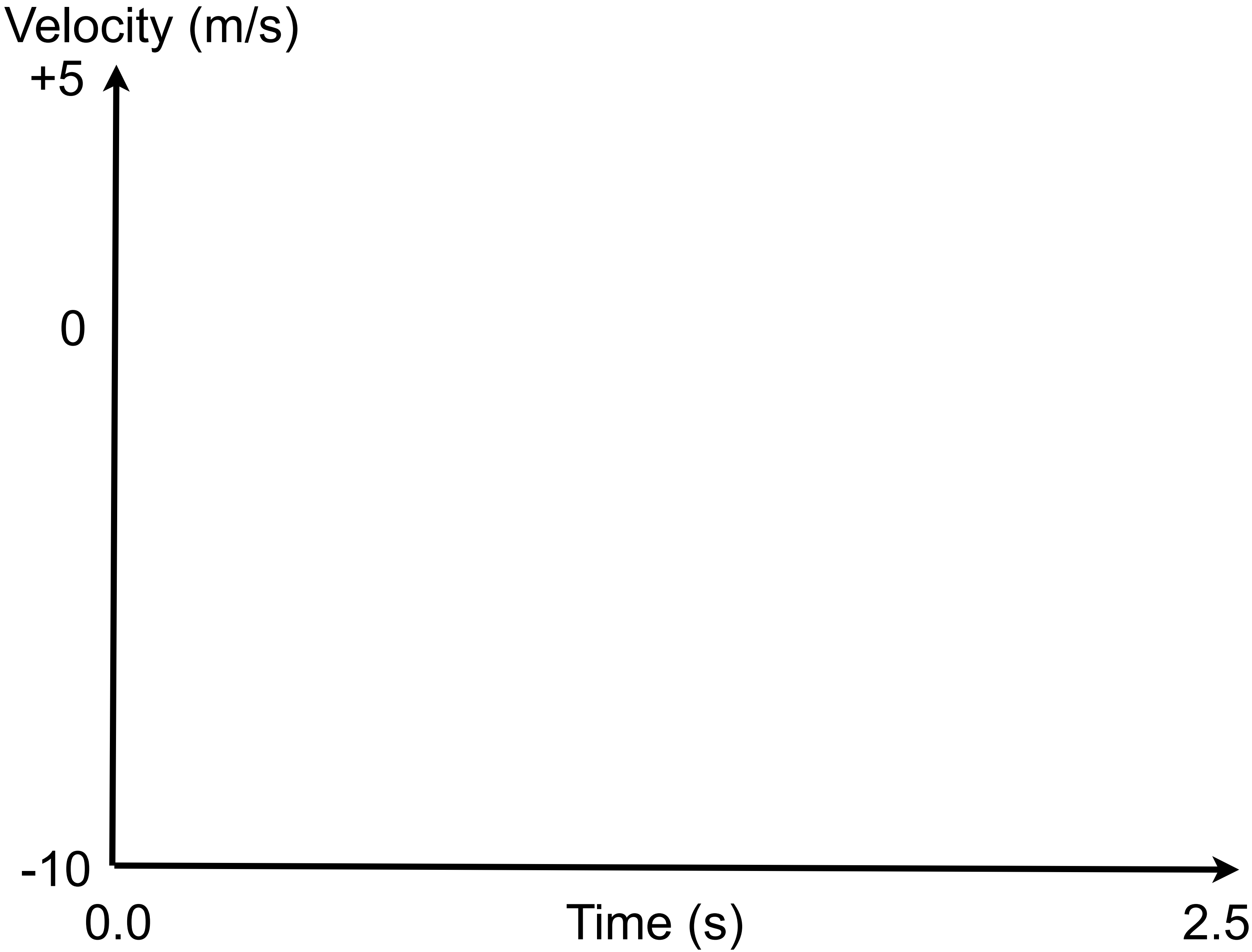
# Basic principle

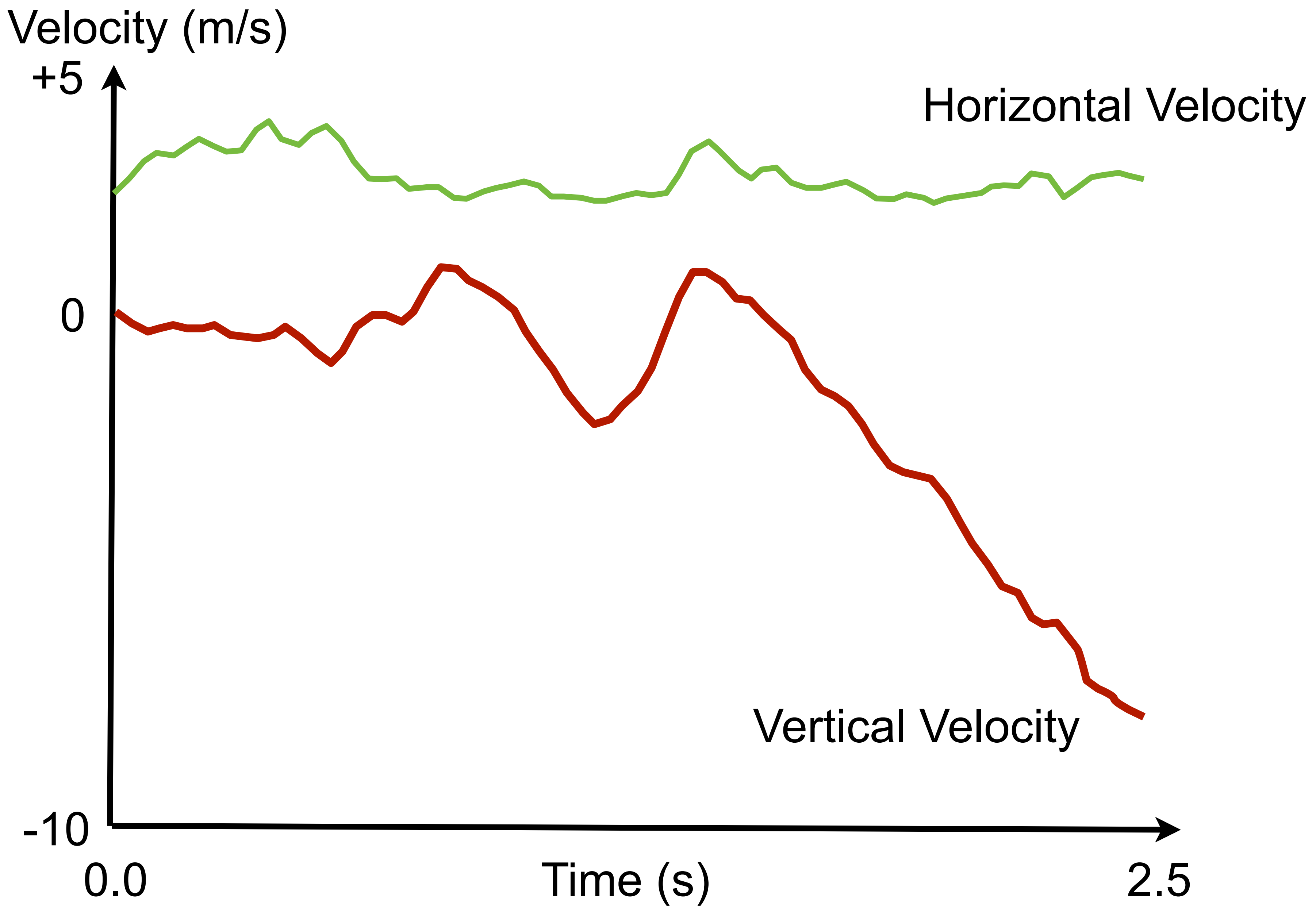
- In the direction in which the (constant) force acts, the object will accelerate at a constant rate.
- In the perpendicular direction, the object's speed will remain constant.
- Does this work in practice?

# What do you think ...

- A graph or her horizontal velocity against time would look like?

- A graph or her vertical velocity against time would look like?


Velocity (m/s)
+5
0
-10
0.0
Time (s)
2.5


Velocity (m/s)
+5
Horizontal Velocity
0
Vertical Velocity
-10
0.0
Time (s)
2.5

# Yes - it does work!

- As in this case air resistance isn't too important.
- The object continues to move at a steady horizontal speed.
- And a uniformly increasing vertical speed.

# In the direction of the force

- Call the position of the object s. F = m a can be written:

$$F = m\frac{d^2s}{dt^2}$$

so

$$\frac{d^2s}{dt^2} = \frac{F}{m} = a$$

Integrate

$$\frac{ds}{dt} = v = at + v_0$$

where $v_0$ is a constant (and is equal to the speed at time 0)

# Integrate again

$$s = v_0 t + \frac{1}{2} a t^2 + s_0$$

where $s_0$ is the position at time zero

# (another useful equation)

- If you don't care about time, combine

$$\frac{ds}{dt} = v = at + v_0 \text{ with } s = v_0 t + \frac{1}{2}at^2 + s_0$$

to eliminate t. From the first, rearrange to get

$$t = \frac{V - V_0}{a}$$

substitute this into the second equation...

$$s - s_0 = v_0 \left( \frac{v - v_0}{a} \right) + \frac{1}{2} a \left( \frac{v - v_0}{a} \right)^2$$

Multiply both sides by 2a and multiply out the bracket

$$2a(s - s_0) = 2vv_0 - 2v_0^2 + v^2 - 2vv_0 + v_0^2$$

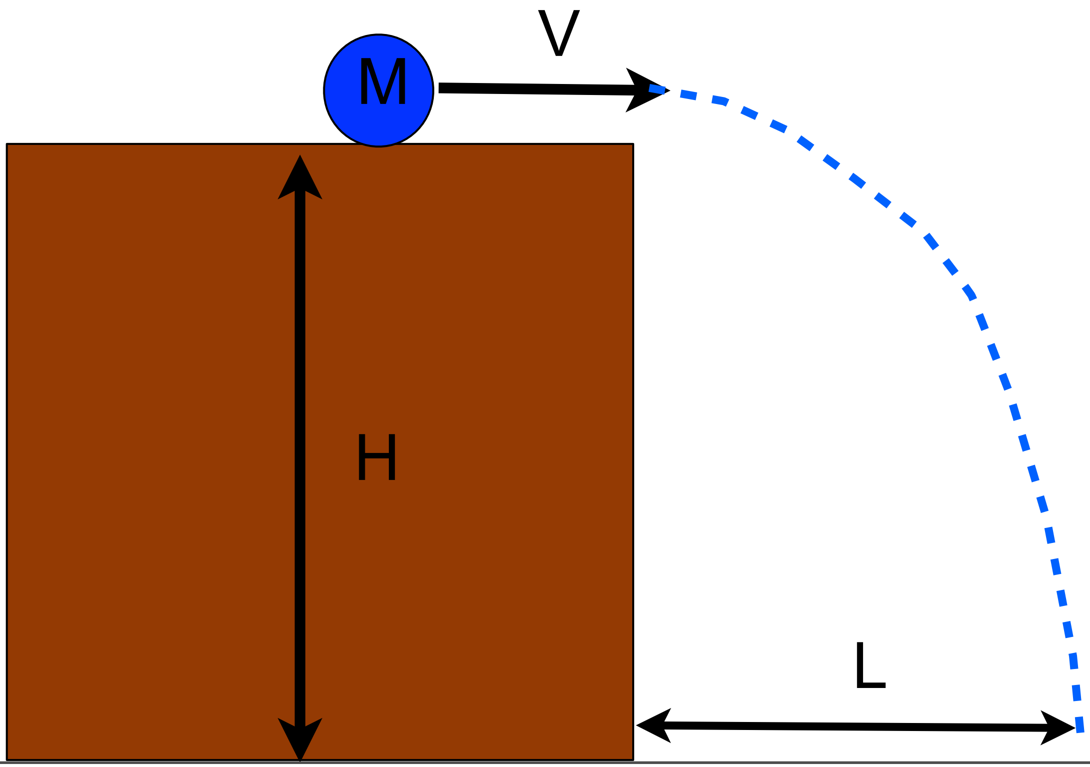
Simplify

$$2a(s - s_0) = v^2 - v_0^2$$

All of which should be familiar equations...

# Example

- A ball is knocked sideways off a table. How far from the base of the table does it land?



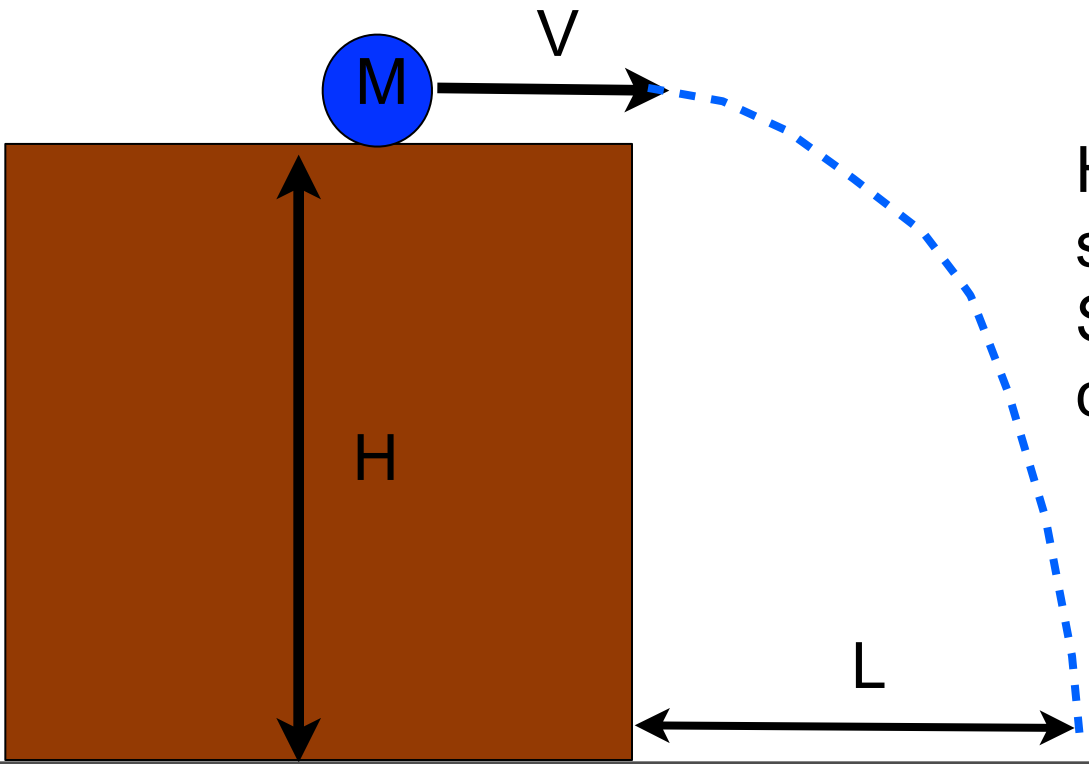

Horizontally, the velocity will remain constant Vertically, it will increase with an acceleration of g until it hits the ground.

Do the vertical equation to work out how long until it hits the ground. It starts off with no vertical velocity, acceleration is g, and travels a distance H. So use $s = v_0 t + \frac{1}{2}at^2$

$$H = \frac{1}{2}gt^2$$

as $v_0$ is zero and the acceleration is g. Solve for t.

$$t = \sqrt{\frac{2H}{g}}$$




How far will it move sideways in this time? Sideways speed is a constant V, so

$$L = Vt = V\sqrt{\frac{2H}{g}}$$

# Key points from this lecture

- Momentum is what keeps things moving in a straight line.
- You must apply a force to change the momentum.
- If you know the force you can work out the motion, and vice versa.
- This usually needs a computer, but in special cases (like uniform force) you can do it analytically.